

QFT as pilot-wave theory of particle creation and destruction

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Abstract

States in quantum field theory (QFT) are represented by many-particle wave functions, such that a state describing n particles depends on n spacetime positions. Since a general state is a superposition of states with different numbers of particles, the wave function lives in the configuration space identified with a product of an infinite number of 4-dimensional Minkowski spacetimes. The squared absolute value of the wave function is interpreted as the probability density in the configuration space, from which the standard probabilistic predictions of QFT can be recovered. Such a formulation and probabilistic interpretation of QFT allows to interpret the wave function as a pilot wave that describes deterministic particle trajectories, which automatically includes a deterministic and continuous description of particle creation and destruction. In particular, when the conditional wave function associated with a quantum measurement ceases to depend on one of the spacetime coordinates, then the 4-velocity of the corresponding particle vanishes, describing a trajectory that stops at a particular point in spacetime. In a more general situation a dependence on this spacetime coordinate is negligibly small but not strictly zero, in which case the trajectory does not stop but the measuring apparatus still behaves as if this particle has been destroyed.

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1 Introduction

The Bohmian interpretation of nonrelativistic quantum mechanics (QM) [1, 2, 3, 4] is the best known and most successful attempt to explain quantum phenomena in terms

of “hidden variables”, that is, objective properties of the system that are well defined even in the absence of measurements. According to this interpretation, particles always have continuous and deterministic trajectories in spacetime, while all quantum uncertainties are an artefact of the ignorance of the initial particle positions. The wave function plays an auxiliary role, by acting as a pilot wave that determines motions of particles for given initial positions.

Yet, in its current form, the Bohmian interpretation is not without difficulties. An important nontrivial issue is to make the Bohmian interpretation of many-particle systems compatible with special relativity. Manifestly relativistic-covariant Bohmian equations of motion of many-particle systems have been proposed in [5] and further studied in [6, 7], but for a long time it has not been known how to associate probabilistic predictions with such relativistic-covariant equations of motion. Recently, a progress has been achieved by realizing that the *a priori* probability density $\rho(\mathbf{x}, t) \propto |\psi(\mathbf{x}, t)|^2$ of a single particle at the space-position \mathbf{x} at time t should *not* be interpreted as a probability density in space satisfying $\int d^3x \rho(\mathbf{x}, t) = 1$, but as a probability density in *spacetime* satisfying $\int d^3x dt \rho(\mathbf{x}, t) = 1$ [8, 9]. The usual probabilistic interpretation in space is then recovered as a conditional probability, corresponding to the case in which the time of detection has been observed. For an n -particle wave function it generalizes to $\int d^4x_1 \cdots \int d^4x_n \rho(x_1, \cdots, x_n) = 1$, where $\rho(x_1, \cdots, x_n) \propto |\psi(x_1, \cdots, x_n)|^2$ and $x_a \equiv (\mathbf{x}_a, t_a)$. As shown in [8, 9], such a manifestly relativistic-invariant probabilistic interpretation is compatible with the manifestly relativistic-covariant Bohmian equations of motion. In this paper we confirm this compatibility, through a more careful discussion in Appendix B.

The main remaining problem is how to make the Bohmian interpretation compatible with quantum field theory (QFT), which predicts that particles can be created and destructed. How to make a theory that describes deterministic continuous trajectories compatible with the idea that a trajectory may have a singular point at which the trajectory begins or ends? One possibility is to explicitly break the rule of continuous deterministic evolution, by adding an additional equation that specifies stochastic breaking of the trajectories [10, 11]. Another possibility is to introduce an additional continuously and deterministically evolving hidden variable that specifies effectivity of each particle trajectory [12, 13]. However, both possibilities seem rather artificial and contrived. A more elegant possibility is to replace pointlike particles by extended strings, in which case the Bohmian equation of motion automatically contains a continuous deterministic description of particle creation and destruction as string splitting [8]. However, string theory is not yet an experimentally confirmed theory, so it would be much more appealing if Bohmian mechanics could be made consistent without strings.

The purpose of this paper is to generalize the relativistic-covariant Bohmian interpretation of relativistic QM [9] describing a fixed number of particles, to relativistic QFT that describes systems in which the number of particles may change. It turns out that the natural Bohmian equations of motion for particles described by QFT automatically describe their creation and destruction, without need to add any additional structure to the theory. In particular, the new artificial structures that has been added in [10, 11] or [12, 13] turn out to be completely unnecessary.

The main new ideas of this paper are introduced in Sec. 2 in a non-technical and intuitive way. This section serves as a motivation for studying the technical details developed in the subsequent sections, but a reader not interested in technical details may be satisfied to read this section only.

Sec. 3 presents in detail several new and many not widely known conceptual and technical results in standard QFT that do not depend on the interpretation of quantum theory. As such, this section may be of interest even for readers not interested in interpretations of quantum theory. The main purpose of this section, however, is to prepare the theoretical framework needed for the physical interpretation studied in the next section.

Sec. 4 finally deals with the physical interpretation. The general probabilistic interpretation and its relation to the usual probabilistic rules in practical applications of QFT is discussed first, while a detailed discussion of the interpretation in terms of deterministic particle trajectories (already indicated in Sec. 2) is delegated to the final part of this section.

Finally, the conclusions are drawn in Sec. 5.

In the paper we use units $\hbar = c = 1$ and the metric signature $(+, -, -, -)$.

2 Main ideas

In this section we formulate our main ideas in a casual and mathematically non-rigorous way, with the intention to develop an intuitive understanding of our results, and to motivate the formal developments that will be presented in the subsequent sections.

As a simple example, consider a QFT state of the form

$$|\Psi\rangle = |1\rangle + |2\rangle, \tag{1}$$

which is a superposition of a 1-particle state $|1\rangle$ and a 2-particle state $|2\rangle$. For example, it may represent an unstable particle for which we do not know if it has already decayed into 2 new particles (in which case it is described by $|2\rangle$) or has not decayed yet (in which case it is described by $|1\rangle$). However, it is known that one always observes either one unstable particle (the state $|1\rangle$) or two decay products (the state $|2\rangle$). One never observes the superposition (1). Why?

To answer this question, let us try with a Bohmian approach. One can associate a 1-particle wave function $\Psi_1(x_1)$ with the state $|1\rangle$ and a 2-particle wave function $\Psi_2(x_2, x_3)$ with the state $|2\rangle$, where x_A is the spacetime position x_A^μ , $\mu = 0, 1, 2, 3$, of the particle labeled by $A = 1, 2, 3$. Then the state (1) is represented by a superposition

$$\Psi(x_1, x_2, x_3) = \Psi_1(x_1) + \Psi_2(x_2, x_3). \tag{2}$$

However, the Bohmian interpretation of such a superposition will describe *three* particle trajectories. On the other hand, we should observe either one or two particles, not three particles. How to explain that?

To understand it intuitively, we find it instructive to first understand an analogous but much simpler problem in Bohmian mechanics. Consider a single non-relativistic

particle moving in 3 dimensions. Its wave function is $\psi(\mathbf{x})$, where $\mathbf{x} = \{x^1, x^2, x^3\}$ and the time-dependence is suppressed. Now let us assume that the particle can be observed to move either along the x^1 direction (in which case it is described by $\psi_1(x^1)$) or on the x^2 - x^3 plane (in which case it is described by $\psi_2(x^2, x^3)$). In other words, the particle is observed to move either in one dimension or two dimensions, but never in all three dimensions. But if we do not know which of these two possibilities will be realized, we describe the system by a superposition

$$\psi(x^1, x^2, x^3) = \psi_1(x^1) + \psi_2(x^2, x^3). \quad (3)$$

However, the Bohmian interpretation of the superposition (3) will lead to a particle that moves in all three dimensions. On the other hand, we should observe that the particle moves either in one dimension or two dimensions. The formal analogy with the many-particle problem above is obvious.

Fortunately, it is very well known how to solve this analogous problem involving one particle that should move either in one or two dimensions. The key is to take into account the properties of the *measuring apparatus*. If it is true that one always observes that the particle moves either in one or two dimensions, then the total wave function describing the entanglement between the measured particle and the measuring apparatus is not (3) but

$$\psi(\mathbf{x}, y) = \psi_1(x^1)E_1(y) + \psi_2(x^2, x^3)E_2(y), \quad (4)$$

where y is a position-variable that describes the configuration of the measuring apparatus. The wave functions $E_1(y)$ and $E_2(y)$ do not overlap. Hence, if y takes a value Y in the support of E_2 , then this value is not in the support of E_1 , i.e., $E_1(Y) = 0$. Consequently, the motion of the measured particle is described by the conditional wave function [14] $\psi_2(x^2, x^3)E_2(Y)$. The effect is the same as if (3) collapsed to $\psi_2(x^2, x^3)$.

Now essentially the same reasoning can be applied to the superposition (2). If the number of particles is measured, then instead of (2) we actually have a wave function of the form

$$\Psi(x_1, x_2, x_3, y) = \Psi_1(x_1)E_1(y) + \Psi_2(x_2, x_3)E_2(y). \quad (5)$$

The detector wave functions $E_1(y)$ and $E_2(y)$ do not overlap. Hence, if the particle counter is found in the state E_2 , then the measured system originally described by (2) is effectively described by $\Psi_2(x_2, x_3)$.

Now, what happens with the particle having the spacetime position x_1 ? In general, its motion in spacetime may be expected to be described by the relativistic Bohmian equation of motion [5, 6, 7]

$$\frac{dX_1^\mu(s)}{ds} = \frac{i\Psi^* \overleftrightarrow{\partial}_1^\mu \Psi}{\Psi^* \Psi}, \quad (6)$$

where s is an auxiliary scalar parameter along the trajectory. However, in our case the effective wave function does not depend on x_1 , i.e., the derivatives in (6) vanish. Consequently, all 4 components of the 4-velocity (6) are zero. The particle does not change its spacetime position X_1^μ . It is an object without an extension not only in

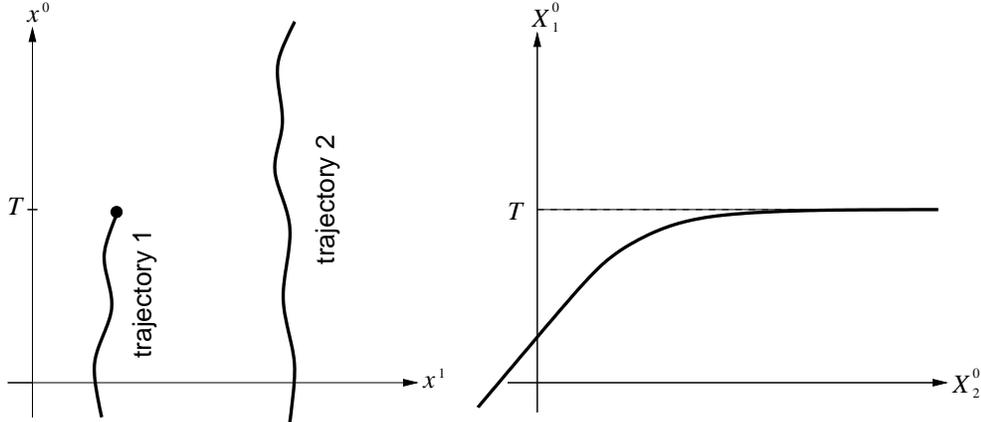


Figure 1: *Left*: The destruction of particle 1 and survival of particle 2, as seen in spacetime. The dot on trajectory 1 denotes a singular point of destruction at $x^0 = T$. *Right*: The same process as seen on the x_1^0 - x_2^0 plane of the configuration space. Instead of a singular point, we have a continuous curve that asymptotically approaches the dashed line $x_1^0 = T$.

space, but also in time. It can be thought of as a pointlike particle that exists only at one instant of time X_1^0 . It lives too short to be detected. Effectively, this particle behaves as if it did not exist at all.

Now consider a more realistic variation of the measuring procedure, taking into account the fact that the measured particles become entangled with the measuring apparatus at some finite time T . Before that, the wave function of the measured particles is really well described by (2). Thus, before the interaction with the measuring apparatus, all 3 particles described by (2) have continuous trajectories in spacetime. All 3 particles exist. But at time T , the total wave function significantly changes. Either (i) y takes a value from the support of E_2 in which case dX_1^μ/ds becomes zero, or (ii) y takes a value from the support of E_1 in which case dX_2^μ/ds and dX_3^μ/ds become zero. After time T , either the particle 1 does not longer change its spacetime position, or the particles 2 and 3 do not longer change their spacetime positions. The effect is the same as if the particle 1 or the particles 2 and 3 do not exist for times $t > T$. In essence, this is how relativistic Bohmian interpretation describes the particle destruction. In order for this mechanism to work, we see that it is essential that each particle possesses not only its own space coordinate \mathbf{x}_A , but also its own time coordinate x_A^0 .

The corresponding particle trajectories are illustrated by Fig. 1. The picture on the left shows the trajectories of particles 1 and 2 in spacetime, for the case in which the particle 1 is destroyed at time T . The trajectory of the destroyed particle looks discontinuous. However, the trajectories of all particles are described by continuous functions $X_A^\mu(s)$ with a common parameter s , so the set of all 3 trajectories (because $A = 1, 2, 3$) in the 4-dimensional spacetime can be viewed as a single continuous trajectory in the $3 \cdot 4 = 12$ dimensional configuration space. The picture on the right of Fig. 1 demonstrates the continuity on the x_1^0 - x_2^0 plane of the configuration space.

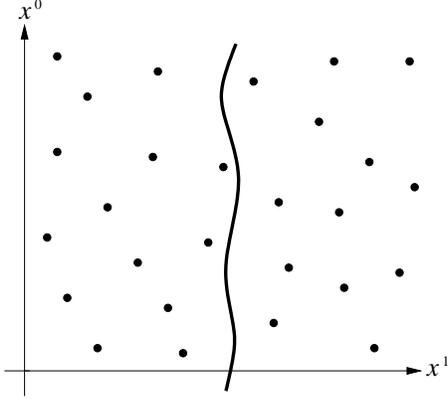


Figure 2: A “real” particle (having non-zero 4-velocity) surrounded by a sea of “vacuum” particles (having zero 4-velocities).

One may object that the mechanism above works only in a very special case in which the absence of the overlap between $E_1(y)$ and $E_2(y)$ is *exact*. In a more realistic situation this overlap is negligibly small, but not exactly zero. In such a situation neither of the particles will have exactly zero 4-velocity. Consequently, neither of the particles will be really destroyed. Nevertheless, the measuring apparatus will still behave as if some particles have been destroyed. For example, if y takes value Y for which $E_1(Y) \ll E_2(Y)$, then for all practical purposes the measuring apparatus behaves as if the wave function collapsed to the second term in (5). The particles with positions X_2 and X_3 also behave in that way. Therefore, even though the particle with the position X_1 is not really destroyed, an effective wave-function collapse still takes place. The influence of the particle with the position X_1 on the measuring apparatus described by Y is negligible, which is effectively the same as if this particle has been destroyed.

Of course, the interaction with the measuring apparatus is not the only mechanism that may induce destruction of particles. Any interaction with the environment may do that. (That is why we use the letter E to denote the states of the measuring apparatus.) Or more generally, any interactions among particles may induce not only particle destruction, but also particle creation. Whenever the wave function $\Psi(x_1, x_2, x_3, x_4, \dots)$ does not really vary (or when this variation is negligible) with some of x_A for some range of values of x_A , then at the edge of this range a trajectory of the particle A may exhibit true (or apparent) creation or destruction.

In general, a QFT state may be a superposition of n -particle states with n ranging from 0 to ∞ . Thus, $\Psi(x_1, x_2, x_3, x_4, \dots)$ should be viewed as a function that lives in the space of infinitely many coordinates x_A , $A = 1, 2, 3, 4, \dots, \infty$. In particular, the 1-particle wave function $\Psi_1(x_1)$ should be viewed as a function $\Psi_1(x_1, x_2, \dots)$ with the property $\partial_A^\mu \Psi_1 = 0$ for $A = 2, 3, \dots, \infty$. It means that any wave function in QFT describes an infinite number of particles, even if most of them have zero 4-velocity. As we have already explained, particles with zero 4-velocity are dots in spacetime. The initial spacetime position of any particle may take any value, with the probability proportional to $|\Psi(x_1, x_2, \dots)|^2$. Thus, the Bohmian particle trajectories associated

with the 1-particle wave function $\Psi_1(x_1, x_2, \dots)$ take a form as in Fig. 2. In addition to one continuous particle trajectory, there is also an infinite number of “vacuum” particles which live for an infinitesimally short time.

It is intuitively clear that a particle that lives for an infinitesimally short time is not observable. However, we have an infinite number of such particles, so could their overall effect be comparable, or even overwhelming, with respect to a finite number of “real” particles that live for a finite time? There is a simple intuitive argument that such an effect should not be expected. The number of “vacuum” particles is equal to the cardinal number of the set on natural numbers, denoted by \aleph_0 . This set has a measure zero with respect to a continuous trajectory, because a continuous trajectory corresponds to a set of real numbers, the cardinal number of which is 2^{\aleph_0} [15]. Intuitively, the number of points on a single continuous trajectory is infinitely times larger than the number of points describing the “vacuum” particles. Consequently, the contribution of the “vacuum” particles to any measurable effect is expected to be negligible.

The purpose of the rest of the paper is to further elaborate the ideas presented in this section and to put them into a more precise framework.

3 Interpretation-independent aspects of QFT

3.1 Measurement in QFT as entanglement with the environment

Let $\{|i\rangle\}$ be some orthonormal basis of 1-particle states. A general normalized 1-particle state is

$$|\Psi_1\rangle = \sum_i c_i |i\rangle, \quad (7)$$

where the normalization condition implies $\sum_i |c_i|^2 = 1$. From the basis $\{|i\rangle\}$ one can construct the n -particle basis $\{|i_1, \dots, i_n\rangle\}$, where

$$|i_1, \dots, i_n\rangle = S_{\{i_1, \dots, i_n\}} |i_1\rangle \cdots |i_n\rangle. \quad (8)$$

Here $S_{\{i_1, \dots, i_n\}}$ denotes the symmetrization over all $\{i_1, \dots, i_n\}$ for bosons, or antisymmetrization for fermions. The most general state in QFT describing these particles can be written as

$$|\Psi\rangle = c_0 |0\rangle + \sum_{n=1}^{\infty} \sum_{i_1, \dots, i_n} c_{n; i_1, \dots, i_n} |i_1, \dots, i_n\rangle, \quad (9)$$

where the vacuum $|0\rangle$ is also introduced. Now the normalization condition implies $|c_0|^2 + \sum_{n=1}^{\infty} \sum_{i_1, \dots, i_n} |c_{n; i_1, \dots, i_n}|^2 = 1$.

Now let us assume that the number of particles is measured. It implies that the particles become entangled with the environment, such that the total state describing both the measured particles and the environment takes the form

$$|\Psi\rangle_{\text{total}} = c_0 |0\rangle |E_0\rangle \quad (10)$$

$$+ \sum_{n=1}^{\infty} \sum_{i_1, \dots, i_n} c_{n; i_1, \dots, i_n} |i_1, \dots, i_n\rangle |E_{n; i_1, \dots, i_n}\rangle.$$

The environment states $|E_0\rangle$, $|E_{n; i_1, \dots, i_n}\rangle$ are macroscopically distinct. They describe what the observers really observe. When an observer observes that the environment is in the state $|E_0\rangle$ or $|E_{n; i_1, \dots, i_n}\rangle$, then one says that the original measured QFT state is in the state $|0\rangle$ or $|i_1, \dots, i_n\rangle$, respectively. In particular, this is how the number of particles is measured in a state (9) with an uncertain number of particles. The probability that the environment will be found in the state $|E_0\rangle$ or $|E_{n; i_1, \dots, i_n}\rangle$ is equal to $|c_0|^2$ or $|c_{n; i_1, \dots, i_n}|^2$, respectively.

Of course, (9) is not the only way the state $|\Psi\rangle$ can be expanded. In general, it can be expanded as

$$|\Psi\rangle = \sum_{\xi} c_{\xi} |\xi\rangle, \quad (11)$$

where $|\xi\rangle$ are some normalized (not necessarily orthogonal) states that do not need to have a definite number of particles. A particularly important example are coherent states (see, e.g., [16]), which minimize the products of uncertainties of fields and their canonical momenta. Each coherent state is a superposition of states with all possible numbers of particles, including zero. The coherent states are overcomplete and not orthogonal. Yet, the expansion (11) may be an expansion in terms of coherent states $|\xi\rangle$ as well.

Furthermore, the entanglement with the environment does not necessarily need to take the form (10). Instead, it may take a more general form

$$|\Psi\rangle_{\text{total}} = \sum_{\xi} c_{\xi} |\xi\rangle |E_{\xi}\rangle, \quad (12)$$

where $|E_{\xi}\rangle$ are macroscopically distinct. In principle, the interaction with the environment may create the entanglement (12) with respect to any set of states $\{|\xi\rangle\}$. In practice, however, some types of expansions are preferred. This fact can be explained by the theory of decoherence [17], which explains why states of the form of (12) are stable only for some particular sets $\{|\xi\rangle\}$. In fact, depending on details of the interactions with the environment, in most real situations the entanglement takes either the form (10) or the form (12) with coherent states $|\xi\rangle$. Since coherent states minimize the uncertainties of fields and their canonical momenta, they behave very much like classical fields. This explains why experiments in quantum optics can often be better described in terms of fields rather than particles (see, e.g., [16]). In fact, the theory of decoherence can explain under what conditions the coherent-state basis becomes preferred over basis with definite numbers of particles [18, 19].

There is one additional physically interesting class of sets $\{|\xi\rangle\}$. They may be eigenstates of the particle number operator defined with respect to *Bogoliubov transformed* (see, e.g., [20]) creation and destruction operators. Thus, even the vacuum may have a nontrivial expansion of the form

$$|0\rangle = c_0 |0'\rangle + \sum_{n=1}^{\infty} \sum_{i'_1, \dots, i'_n} c_{n; i'_1, \dots, i'_n} |i'_1, \dots, i'_n\rangle, \quad (13)$$

where the prime denotes the n -particle states with respect to the Bogoliubov transformed number operator. In fact, whenever the two definitions of particles are related by a Bogoliubov transformation, the vacuum for one definition of particles is a squeezed state when expressed in terms of particles of the other definition of particles [21]. Thus, if the entanglement with the environment takes the form

$$|\Psi\rangle_{\text{total}} = c_0|0'\rangle|E_0\rangle + \sum_{n=1}^{\infty} \sum_{i'_1, \dots, i'_n} c_{n; i'_1, \dots, i'_n} |i'_1, \dots, i'_n\rangle |E_{n; i'_1, \dots, i'_n}\rangle, \quad (14)$$

then the vacuum (13) may appear as a state with many particles. Indeed, this is expected to occur when the particle detector is accelerated or when a gravitational field is present [20]. The theory of decoherence can explain why the interaction with the environment leads to an entanglement of the form of (14) [22, 23, 24].

Thus, decoherence induced by interaction with the environment can explain why do we observe either a definite number of particles or coherent states that behave very much like classical fields. However, decoherence alone cannot explain why do we observe some particular state of definite number of particles and not some other, or why do we observe some particular coherent state and not some other.

3.2 Free scalar QFT in the particle-position picture

Consider a free scalar hermitian field operator $\hat{\phi}(x)$ satisfying the Klein-Gordon equation

$$\partial^\mu \partial_\mu \hat{\phi}(x) + m^2 \hat{\phi}(x) = 0. \quad (15)$$

The field can be decomposed as

$$\hat{\phi}(x) = \hat{\psi}(x) + \hat{\psi}^\dagger(x), \quad (16)$$

where $\hat{\psi}$ and $\hat{\psi}^\dagger$ can be expanded as

$$\begin{aligned} \hat{\psi}(x) &= \int d^3k f(\mathbf{k}) \hat{a}(\mathbf{k}) e^{-i[\omega(\mathbf{k})x^0 - \mathbf{k}\mathbf{x}]}, \\ \hat{\psi}^\dagger(x) &= \int d^3k f(\mathbf{k}) \hat{a}^\dagger(\mathbf{k}) e^{i[\omega(\mathbf{k})x^0 - \mathbf{k}\mathbf{x}]} \end{aligned} \quad (17)$$

Here

$$\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2} \quad (18)$$

is the k^0 component of the 4-vector $k = \{k^\mu\}$, and $\hat{a}^\dagger(\mathbf{k})$ and $\hat{a}(\mathbf{k})$ are the usual creation and destruction operators, respectively. The function $f(\mathbf{k})$ is a real positive function which we do not specify explicitly because several different choices appear in the literature, corresponding to several different choices of normalization. All subsequent equations will be written in forms that do not depend on this choice.

We define the operator

$$\hat{\psi}_n(x_{n,1}, \dots, x_{n,n}) = d_n S_{\{x_{n,1}, \dots, x_{n,n}\}} \hat{\psi}(x_{n,1}) \cdots \hat{\psi}(x_{n,n}). \quad (19)$$

The symbol $S_{\{x_{n,1}, \dots, x_{n,n}\}}$ denotes the symmetrization, reminding us that the expression is symmetric under the exchange of coordinates $\{x_{n,1}, \dots, x_{n,n}\}$. (Note, however, that the product of operators on the right hand side of (19) is in fact automatically symmetric because the operators $\hat{\psi}(x)$ commute, i.e., $[\hat{\psi}(x), \hat{\psi}(x')] = 0$.) The parameter d_n is a normalization constant determined by the normalization condition that will be specified below. The operator (19) allows us to define n -particle states in the basis of particle spacetime positions, as

$$|x_{n,1}, \dots, x_{n,n}\rangle = \hat{\psi}_n^\dagger(x_{n,1}, \dots, x_{n,n})|0\rangle. \quad (20)$$

All states of the form (20), together with the vacuum $|0\rangle$, form a complete and orthogonal basis in the Hilbert space of physical states.

If $|\Psi_n\rangle$ is an arbitrary (but normalized) n -particle state, then this state can be represented by the n -particle wave function

$$\psi_n(x_{n,1}, \dots, x_{n,n}) = \langle x_{n,1}, \dots, x_{n,n} | \Psi_n \rangle. \quad (21)$$

We also have

$$\langle x_{n,1}, \dots, x_{n,n} | \Psi_{n'} \rangle = 0 \text{ for } n \neq n'. \quad (22)$$

We choose the normalization constant d_n in (19) such that the following normalization condition is satisfied

$$\int d^4x_{n,1} \cdots \int d^4x_{n,n} |\psi_n(x_{n,1}, \dots, x_{n,n})|^2 = 1. \quad (23)$$

However, this implies that the wave functions $\psi_n(x_{n,1}, \dots, x_{n,n})$ and $\psi_{n'}(x_{n',1}, \dots, x_{n',n'})$, with different values of n and n' , are normalized in different spaces. On the other hand, we want these wave functions to live in the same space, such that we can form superpositions of wave functions describing different numbers of particles. To accomplish this, we define

$$\Psi_n(x_{n,1}, \dots, x_{n,n}) = \sqrt{\frac{\mathcal{V}^{(n)}}{\mathcal{V}}} \psi_n(x_{n,1}, \dots, x_{n,n}), \quad (24)$$

where

$$\mathcal{V}^{(n)} = \int d^4x_{n,1} \cdots \int d^4x_{n,n}, \quad (25)$$

$$\mathcal{V} = \prod_{n=1}^{\infty} \mathcal{V}^{(n)}, \quad (26)$$

are volumes of the corresponding configuration spaces. In particular, the wave function of the vacuum is

$$\Psi_0 = \frac{1}{\sqrt{\mathcal{V}}}. \quad (27)$$

This provides that all wave functions are normalized in the same configuration space as

$$\int \mathcal{D}\vec{x} |\Psi_n(x_{n,1}, \dots, x_{n,n})|^2 = 1, \quad (28)$$

where we use the notation

$$\vec{x} = (x_{1,1}, x_{2,1}, x_{2,2}, \dots), \quad (29)$$

$$\mathcal{D}\vec{x} = \prod_{n=1}^{\infty} \prod_{a_n=1}^n d^4 x_{n,a_n}. \quad (30)$$

Note that the physical Hilbert space does not contain non-symmetrized states, such as a 3-particle state $|x_{1,1}\rangle|x_{2,1}, x_{2,2}\rangle$. It also does not contain states that do not satisfy (18). Nevertheless, the notation can be further simplified by introducing an extended kinematic Hilbert space that contains such unphysical states as well. Every physical state can be viewed as a state in such an extended Hilbert space, although most of the states in the extended Hilbert space are not physical. In this extended space it is convenient to denote the pair of labels (n, a_n) by a single label A . Hence, (29) and (30) are now written as

$$\vec{x} = (x_1, x_2, x_3, \dots), \quad (31)$$

$$\mathcal{D}\vec{x} = \prod_{A=1}^{\infty} d^4 x_A. \quad (32)$$

Similarly, (26) with (25) is now written as

$$\mathcal{V} = \int \prod_{A=1}^{\infty} d^4 x_A. \quad (33)$$

The particle-position basis of this extended space is denoted by $|\vec{x}\rangle$ (which should be distinguished from $|\vec{x}\rangle$ which would denote a symmetrized state of an infinite number of physical particles). Such a basis allows us to write the physical wave function (24) as a wave function on the extended space

$$\Psi_n(\vec{x}) = (\vec{x}|\Psi_n\rangle). \quad (34)$$

Now (28) takes a simpler form

$$\int \mathcal{D}\vec{x} |\Psi_n(\vec{x})|^2 = 1. \quad (35)$$

The normalization (35) corresponds to the normalization in which the unit operator on the extended space is

$$1 = \int \mathcal{D}\vec{x} |\vec{x}\rangle\langle\vec{x}|, \quad (36)$$

while the scalar product is

$$\langle\vec{x}|\vec{x}'\rangle = \delta(\vec{x} - \vec{x}'), \quad (37)$$

with $\delta(\vec{x} - \vec{x}') \equiv \prod_{A=1}^{\infty} \delta^4(x_A - x'_A)$. A general physical state can be written as

$$\Psi(\vec{x}) = \langle\vec{x}|\Psi\rangle = \sum_{n=0}^{\infty} c_n \Psi_n(\vec{x}). \quad (38)$$

It is also convenient to write this as

$$\Psi(\vec{x}) = \sum_{n=0}^{\infty} \tilde{\Psi}_n(\vec{x}), \quad (39)$$

where the tilde denotes a wave function that is not necessarily normalized. The total wave function is normalized, in the sense that

$$\int \mathcal{D}\vec{x} |\Psi(\vec{x})|^2 = 1, \quad (40)$$

implying

$$\sum_{n=0}^{\infty} |c_n|^2 = 1. \quad (41)$$

Next, we introduce the operator

$$\square = \sum_{A=1}^{\infty} \partial_A^\mu \partial_{A\mu}. \quad (42)$$

From the equations above (see, in particular, (15)-(21)), it is easy to show that $\Psi_n(\vec{x})$ satisfies

$$\square \Psi_n(\vec{x}) + nm^2 \Psi_n(\vec{x}) = 0. \quad (43)$$

Introducing a hermitian number-operator \hat{N} with the property

$$\hat{N} \Psi_n(\vec{x}) = n \Psi_n(\vec{x}), \quad (44)$$

one finds that a general physical state (38) satisfies the generalized Klein-Gordon equation

$$\square \Psi(\vec{x}) + m^2 \hat{N} \Psi(\vec{x}) = 0. \quad (45)$$

We also introduce the generalized Klein-Gordon current

$$J_A^\mu(\vec{x}) = \frac{i}{2} \Psi^*(\vec{x}) \overleftrightarrow{\partial}_A^\mu \Psi(\vec{x}). \quad (46)$$

From (45) one finds that, in general, this current is not conserved

$$\sum_{A=1}^{\infty} \partial_{A\mu} J_A^\mu(\vec{x}) = J(\vec{x}), \quad (47)$$

where

$$J(\vec{x}) = -\frac{i}{2} m^2 \Psi^*(\vec{x}) \overleftrightarrow{\hat{N}} \Psi(\vec{x}), \quad (48)$$

and $\Psi' \overleftrightarrow{\hat{N}} \Psi \equiv \Psi'(\hat{N}\Psi) - (\hat{N}\Psi')\Psi$. From (48) we see that the current is conserved in two special cases: (i) when $\Psi = \Psi_n$ (a state with a definite number of physical particles), or (ii) when $m^2 = 0$ (any physical state of massless particles).

In the extended Hilbert space it is also useful to introduce the momentum picture through the Fourier transforms. We define

$$\Psi_{\vec{k}}(\vec{x}) = \sqrt{\frac{(2\pi)^{4\aleph_0}}{\mathcal{V}}} (\vec{x}|\vec{k}) = \frac{e^{-i\vec{k}\vec{x}}}{\sqrt{\mathcal{V}}}, \quad (49)$$

where $\vec{k}\vec{x} \equiv \sum_{A=1}^{\infty} k_{A\mu} x_A^\mu$ and $\aleph_0 = \infty$ corresponds to the number of different values of the label A . In the basis of momentum eigenstates $|\vec{k}\rangle$ we have

$$1 = \int \mathcal{D}\vec{k} |\vec{k}\rangle\langle\vec{k}|, \quad (50)$$

$$\langle\vec{k}|\vec{k}'\rangle = \delta(\vec{k} - \vec{k}'). \quad (51)$$

It is easy to check that the normalizations as above make the Fourier transform

$$\Psi(\vec{k}) = \langle\vec{k}|\Psi\rangle = \int \mathcal{D}\vec{x} (\vec{k}|\vec{x})(\vec{x}|\Psi) \quad (52)$$

and its inverse

$$\Psi(\vec{x}) = (\vec{x}|\Psi) = \int \mathcal{D}\vec{k} (\vec{x}|\vec{k})(\vec{k}|\Psi) \quad (53)$$

consistent. We can also introduce the momentum operator

$$\hat{p}_{A\mu} = i\partial_{A\mu}. \quad (54)$$

The wave function (49) is the momentum eigenstate

$$\hat{p}_{A\mu} \Psi_{\vec{k}}(\vec{x}) = k_{A\mu} \Psi_{\vec{k}}(\vec{x}). \quad (55)$$

In particular, the wave function of the physical vacuum is given by (27), so

$$\hat{p}_{A\mu} \Psi_0(\vec{x}) = 0. \quad (56)$$

We see that (27) can also be written as

$$\Psi_0(\vec{x}) = \frac{e^{-i\vec{0}\vec{x}}}{\sqrt{\mathcal{V}}}, \quad (57)$$

showing that the physical vacuum can also be represented as

$$|0\rangle = |\vec{k} = \vec{0}\rangle. \quad (58)$$

Intuitively, it means that the vacuum can be thought of as a state with an infinite number of particles, all of which have vanishing 4-momentum. Similarly, an n -particle state can be thought of as a state with an infinite number of particles, where only n of them have a non-vanishing 4-momentum.

Finally, let us rewrite some of the main results of this (somewhat lengthy) subsection in a form that will be suitable for a generalization in the next subsection. A general physical state can be written in the form

$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n |\Psi_n\rangle = \sum_{n=0}^{\infty} |\tilde{\Psi}_n\rangle. \quad (59)$$

The corresponding unnormalized n -particle wave functions are

$$\tilde{\psi}_n(x_{n,1}, \dots, x_{n,n}) = \langle 0 | \hat{\psi}_n(x_{n,1}, \dots, x_{n,n}) | \Psi \rangle. \quad (60)$$

There is a well-defined transformation

$$\tilde{\psi}_n(x_{n,1}, \dots, x_{n,n}) \rightarrow \tilde{\Psi}_n(\vec{x}) \quad (61)$$

from the physical Hilbert space to the extended Hilbert space, so that the general state (59) can be represented by a single wave function

$$\Psi(\vec{x}) = \sum_{n=0}^{\infty} c_n \Psi_n(\vec{x}) = \sum_{n=0}^{\infty} \tilde{\Psi}_n(\vec{x}). \quad (62)$$

3.3 Generalization to the interacting QFT

In this subsection we discuss the generalization of the results of the preceding subsection to the case in which the field operator $\hat{\phi}$ does not satisfy the free Klein-Gordon equation (15). For example, if the classical action is

$$S = \int d^4x \left[\frac{1}{2} (\partial^\mu \phi) (\partial_\mu \phi) - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4 \right], \quad (63)$$

then (15) generalizes to

$$\partial^\mu \partial_\mu \hat{\phi}_H(x) + m^2 \hat{\phi}_H(x) + \lambda \hat{\phi}_H^3(x) = 0, \quad (64)$$

where $\hat{\phi}_H(x)$ is the field operator in the Heisenberg picture. (From this point of view, the operator $\hat{\phi}(x)$ defined by (16) and (17) and satisfying the free Klein-Gordon equation (15) is the field operator in the interaction (Dirac) picture.) Thus, instead of (60) now we have

$$\tilde{\psi}_n(x_{n,1}, \dots, x_{n,n}) = \langle 0 | \hat{\psi}_{nH}(x_{n,1}, \dots, x_{n,n}) | \Psi \rangle, \quad (65)$$

where $|\Psi\rangle$ and $|0\rangle$ are states in the Heisenberg picture. Assuming that (65) has been calculated (we shall see below how in practice it can be done), the rest of the job is straightforward. One needs to make the transformation (61) in the same way as in the free case, which leads to an interacting variant of (62)

$$\Psi(\vec{x}) = \sum_{n=0}^{\infty} \tilde{\Psi}_n(\vec{x}). \quad (66)$$

The wave function (66) encodes the complete information about the properties of the interacting system.

Now let us see how (65) can be calculated in practice. Any operator $\hat{O}_H(t)$ in the Heisenberg picture depending on a single time-variable t can be written in terms of operators in the interaction picture as

$$\hat{O}_H(t) = \hat{U}^\dagger(t) \hat{O}(t) \hat{U}(t), \quad (67)$$

where

$$\hat{U}(t) = T e^{-i \int_{t_0}^t dt' \hat{H}_{\text{int}}(t')}, \quad (68)$$

t_0 is some appropriately chosen “initial” time, T denotes the time ordering, and \hat{H}_{int} is the interaction part of the Hamiltonian expressed as a functional of field operators in the interaction picture (see, e.g., [25]). For example, for the action (63) we have

$$\hat{H}_{\text{int}}(t) = \frac{\lambda}{4} \int d^3x : \hat{\phi}^4(\mathbf{x}, t) :, \quad (69)$$

where $::$ denotes the normal ordering. The relation (67) can also be inverted, leading to

$$\hat{O}(t) = \hat{U}(t) \hat{O}_H(t) \hat{U}^\dagger(t). \quad (70)$$

Thus, the relation (19), which is now valid in the interaction picture, allows us to write an analogous relation in the Heisenberg picture

$$\begin{aligned} \hat{\psi}_{nH}(x_{n,1}, \dots, x_{n,n}) &= d_n S_{\{x_{n,1}, \dots, x_{n,n}\}} \\ &\hat{\psi}_H(x_{n,1}) \cdots \hat{\psi}_H(x_{n,n}), \end{aligned} \quad (71)$$

where

$$\hat{\psi}_H(x_{n,a_n}) = \hat{U}^\dagger(x_{n,a_n}^0) \hat{\psi}(x_{n,a_n}) \hat{U}(x_{n,a_n}^0). \quad (72)$$

By expanding (68) in powers of $\int_{t_0}^t dt' \hat{H}_{\text{int}}$, this allows us to calculate (71) and (65) perturbatively. In (65), the states in the Heisenberg picture $|\Psi\rangle$ and $|0\rangle$ are identified with the states in the interaction picture at the initial time $|\Psi(t_0)\rangle$ and $|0(t_0)\rangle$, respectively.

To demonstrate that such a procedure leads to a physically sensible result, let us see how it works in the special (and more familiar) case of the equal-time wave function. It is given by $\tilde{\psi}_n(x_{n,1}, \dots, x_{n,n})$ calculated at $x_{n,1}^0 = \dots = x_{n,n}^0 \equiv t$. Thus, (65) reduces to

$$\begin{aligned} \tilde{\psi}_n(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; t) &= d_n \langle 0(t_0) | \hat{U}^\dagger(t) \hat{\psi}(\mathbf{x}_{n,1}, t) \hat{U}(t) \\ &\cdots \hat{U}^\dagger(t) \hat{\psi}(\mathbf{x}_{n,n}, t) \hat{U}(t) | \Psi(t_0) \rangle. \end{aligned} \quad (73)$$

Using $\hat{U}(t) \hat{U}^\dagger(t) = 1$ and

$$\hat{U}(t) |\Psi(t_0)\rangle = |\Psi(t)\rangle, \quad \hat{U}(t) |0(t_0)\rangle = |0(t)\rangle, \quad (74)$$

the expression further simplifies

$$\begin{aligned} \tilde{\psi}_n(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; t) &= \\ d_n \langle 0(t) | \hat{\psi}(\mathbf{x}_{n,1}, t) \cdots \hat{\psi}(\mathbf{x}_{n,n}, t) | \Psi(t) \rangle. \end{aligned} \quad (75)$$

In practical applications of QFT in particle physics, one usually calculates the S -matrix, corresponding to the limit $t_0 \rightarrow -\infty$, $t \rightarrow \infty$. For Hamiltonians that conserve energy (such as (69)) this limit provides the stability of the vacuum, i.e., obeys

$$\lim_{t_0 \rightarrow -\infty, t \rightarrow \infty} \hat{U}(t) |0(t_0)\rangle = e^{-i\varphi_0} |0(t_0)\rangle, \quad (76)$$

where φ_0 is some physically irrelevant phase [26]. Essentially, this is because the integrals of the type $\int_{-\infty}^{\infty} dt' \dots$ produce δ -functions that correspond to energy conservation, so the vacuum remains stable because particle creation from the vacuum would violate energy conservation. Thus we have

$$|0(\infty)\rangle = e^{-i\varphi_0}|0(-\infty)\rangle \equiv e^{-i\varphi_0}|0\rangle. \quad (77)$$

The state

$$|\Psi(\infty)\rangle = \hat{U}(\infty)|\Psi(-\infty)\rangle \quad (78)$$

is not trivial, but whatever it is, it has some expansion of the form

$$|\Psi(\infty)\rangle = \sum_{n=0}^{\infty} c_n(\infty)|\Psi_n\rangle, \quad (79)$$

where $c_n(\infty)$ are some coefficients. Plugging (77) and (79) into (75) and recalling (19)-(22), we finally obtain

$$\tilde{\psi}_n(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; \infty) = e^{i\varphi_0} c_n(\infty) \psi_n(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; \infty). \quad (80)$$

This demonstrates the consistency of (65), because (78) should be recognized as the standard description of evolution from $t_0 \rightarrow -\infty$ to $t \rightarrow \infty$ (see, e.g., [25, 26]), showing that the coefficients $c_n(\infty)$ are the same as those described by standard S -matrix theory in QFT. In other words, (65) is a natural many-time generalization of the concept of single-time evolution in interacting QFT.

3.4 Generalization to other types of particles

In Secs. 3.2 and 3.3 we have discussed in detail scalar hermitian fields, corresponding to spinless uncharged particles. In this subsection we briefly discuss how these results can be generalized to any type of fields and the corresponding particles.

In general, fields ϕ carry some additional labels which we collectively denote by l , so we deal with fields ϕ_l . For example, spin-1 field carries a vector index, fermionic spin- $\frac{1}{2}$ field carries a spinor index, non-Abelian gauge fields carry internal indices of the gauge group, etc. Thus Eq. (19) generalizes to

$$\hat{\psi}_{n,L_n}(x_{n,1}, \dots, x_{n,n}) = d_n S_{\{x_{n,1}, \dots, x_{n,n}\}} \hat{\psi}_{l_{n,1}}(x_{n,1}) \cdots \hat{\psi}_{l_{n,n}}(x_{n,n}), \quad (81)$$

where L_n is a collective label $L_n = (l_{n,1}, \dots, l_{n,n})$. The symbol $S_{\{x_{n,1}, \dots, x_{n,n}\}}$ denotes symmetrization (antisymmetrization) over bosonic (fermionic) fields describing the same type of particles. Hence, it is straightforward to make the appropriate generalizations of all results of Secs. 3.2 and 3.3. For example, (39) generalizes to

$$\Psi_{\vec{L}}(\vec{x}) = \sum_{n=0}^{\infty} \sum_{L_n} \tilde{\Psi}_{n,L_n}(\vec{x}), \quad (82)$$

with self-explaining notation.

To further simplify the notation, we introduce the column $\Psi \equiv \{\Psi_{\vec{L}}\}$ and the row $\Psi^\dagger \equiv \{\Psi_{\vec{L}}^*\}$. With this notation, the appropriate generalization of (40) can be written as

$$\int \mathcal{D}\vec{x} \sum_{\vec{L}} \Psi_{\vec{L}}^*(\vec{x}) \Psi_{\vec{L}}(\vec{x}) \equiv \int \mathcal{D}\vec{x} \Psi^\dagger(\vec{x}) \Psi(\vec{x}) = 1. \quad (83)$$

For the case of states that contain fermionic particles, Eq. (83) requires further discussion. As a simple example, consider a 1-particle state describing one electron. In this case, (83) can be reduced to

$$\int d^4x \psi^\dagger(x) \psi(x) = 1, \quad (84)$$

where ψ is a Dirac spinor. In this expression, the quantity $\psi^\dagger\psi$ must transform as a Lorentz scalar. At first sight, it may seem to be in contradiction with the well-known fact that $\psi^\dagger\psi = \bar{\psi}\gamma^0\psi$ transforms as a time-component of a Lorentz vector. However, there is no true contradiction. Let us explain.

The standard derivation that $\bar{\psi}\gamma^\mu\psi$ transforms as a vector [27] starts from the assumption that the matrices γ^μ do not transform under Lorentz transformations, despite of carrying the index μ . However, such an assumption is not necessary. Moreover, in curved spacetime such an assumption is inconsistent [20]. In fact, one is allowed to define the transformations of ψ and γ^μ in an arbitrary way, as long as such transformations do not affect the transformations of measurable quantities, or quantities like $\bar{\psi}\gamma^\mu\psi$ that are closely related to measurable ones. Thus, it is much more natural to deal with a differently defined transformations of γ^μ and ψ , such that γ^μ transforms as a vector and ψ transforms as a scalar under Lorentz transformations of spacetime coordinates [20]. The spinor indices of γ^μ and ψ are then reinterpreted as indices in an internal space. With such redefined transformations, (84) is fully consistent. The details of our transformation conventions are presented in Appendix A.

4 The physical interpretation

4.1 Probabilistic interpretation

In this subsection we adopt and further develop the probabilistic interpretation introduced in [9] (and partially inspired by earlier results [28, 29, 30, 31, 32]). The quantity

$$\mathcal{D}P = \Psi^\dagger(\vec{x}) \Psi(\vec{x}) \mathcal{D}\vec{x} \quad (85)$$

is naturally interpreted as the probability of finding the system in the (infinitesimal) configuration-space volume $\mathcal{D}\vec{x}$ around a point \vec{x} in the configuration space. Indeed, such an interpretation is consistent with our normalization conditions such as (40) and (83). In more physical terms, it gives the joint probability that the particle 1 is found at the spacetime position x_1 , particle 2 at the spacetime position x_2 , etc. Similarly, the Fourier-transformed wave function $\Psi(\vec{k})$ defines the probability $\Psi^\dagger(\vec{k}) \Psi(\vec{k}) \mathcal{D}\vec{k}$,

which is the joint probability that the particle 1 has the 4-momentum k_1 , particle 2 the 4-momentum k_2 , etc.

As a special case, consider an n -particle state $\Psi(\vec{x}) = \Psi_n(\vec{x})$. It really depends only on n spacetime positions $x_{n,1}, \dots, x_{n,n}$. With respect to all other positions x_B , Ψ is a constant. Thus, the probability of various positions x_B does not depend on x_B ; such a particle can be found anywhere and anytime with equal probabilities. There is an infinite number of such particles. Nevertheless, the Fourier transform of such a wave function reveals that the 4-momentum k_B of these particles is necessarily zero; they have neither 3-momentum nor energy. For that reason, such particles can be thought of as “vacuum” particles. In this picture, an n -particle state Ψ_n is thought of as a state describing n “real” particles and an infinite number of “vacuum” particles.

To avoid a possible confusion with the usual notions of vacuum and real particles in QFT, in the rest of the paper we refer to “vacuum” particles as *dead* particles and “real” particles as *live* particles. Or let us be slightly more precise: We say that the particle A is dead if the wave function in the momentum space $\Psi(\vec{k})$ vanishes for all values of k_A except $k_A = 0$. Similarly, we say that the particle A is live if it is not dead.

The properties of live particles associated with the state $\Psi_n(\vec{x})$ can also be represented by the wave function $\psi_n(x_{n,1}, \dots, x_{n,n})$. By averaging over physically uninteresting dead particles, (85) reduces to

$$dP = \psi_n^\dagger(x_{n,1}, \dots, x_{n,n}) \psi_n(x_{n,1}, \dots, x_{n,n}) \times d^4x_{n,1} \cdots d^4x_{n,n}, \quad (86)$$

which involves only live particles. This describes the probability when neither the space positions of detected particles nor times of their detections are known. To relate it with a more familiar probabilistic interpretation of QM, let us consider the special case; let us assume that the first particle is detected at time $x_{n,1}^0$, second particle at time $x_{n,2}^0$, etc. In this case, the detection times are known, so (86) is no longer the best description of our knowledge about the system. Instead, the relevant probability derived from (86) is the *conditional* probability

$$dP_{(3n)} = \frac{\psi_n^\dagger(x_{n,1}, \dots, x_{n,n}) \psi_n(x_{n,1}, \dots, x_{n,n})}{N_{x_{n,1}^0, \dots, x_{n,n}^0}} \times d^3x_{n,1} \cdots d^3x_{n,n}, \quad (87)$$

where

$$N_{x_{n,1}^0, \dots, x_{n,n}^0} = \int \psi_n^\dagger(x_{n,1}, \dots, x_{n,n}) \psi_n(x_{n,1}, \dots, x_{n,n}) \times d^3x_{n,1} \cdots d^3x_{n,n} \quad (88)$$

is the appropriate normalization factor. (For more details regarding the meaning and limitations of (87) in the 1-particle case see Appendix B.) The probability (87) is sometimes also postulated as a fundamental axiom of many-time formulation of QM [33], but here (87) is derived from a more fundamental and more general expression

(86) (which, in turn, is derived from an even more general axiom (85)). An even more familiar expression is obtained by studying a special case of (87) in which $x_{n,1}^0 = \dots = x_{n,n}^0 \equiv t$, so that (87) reduces to

$$dP_{(3n)} = \frac{\psi_n^\dagger(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; t) \psi_n(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; t)}{N_t} \times d^3x_{n,1} \cdots d^3x_{n,n}, \quad (89)$$

where N_t is given by (88) evaluated at $x_{n,1}^0 = \dots = x_{n,n}^0 \equiv t$.

Now let us see how the wave functions representing the states in interacting QFT are interpreted probabilistically. Consider the wave function $\tilde{\psi}_n(x_{n,1}, \dots, x_{n,n})$ given by (65). For example, it may vanish for small values of $x_{n,1}^0, \dots, x_{n,n}^0$, but it may not vanish for their large values. Physically, it means that these particles cannot be detected in the far past (the probability is zero), but that they can be detected in the far future. This is nothing but a probabilistic description of the creation of n particles that have not existed in the far past. Indeed, the results obtained in Sec. 3.3 (see, in particular, (80)) show that such probabilities are consistent with the probabilities of particle creation obtained by the standard S -matrix methods in QFT.

Having developed the probabilistic interpretation, we can also calculate the average values of various quantities. We are particularly interested in average values of the 4-momentum p_A^μ . In general, its average value is

$$\langle p_A^\mu \rangle = \int \mathcal{D}\vec{x} \Psi^\dagger(\vec{x}) \hat{p}_A^\mu \Psi(\vec{x}), \quad (90)$$

where \hat{p}_A^μ is given by (54). If $\Psi(\vec{x}) = \Psi_n(\vec{x})$, then (90) can be reduced to

$$\langle p_{n,a_n}^\mu \rangle = \int d^4x_{n,1} \cdots d^4x_{n,n} \psi_n^\dagger(x_{n,1}, \dots, x_{n,n}) \hat{p}_{n,a_n}^\mu \psi_n(x_{n,1}, \dots, x_{n,n}). \quad (91)$$

Similarly, if the times of detections are known and are all equal to t , then the average space-components of momenta are given by a more familiar expression

$$\langle \mathbf{p}_{n,a_n} \rangle = N_t^{-1} \int d^3x_{n,1} \cdots d^3x_{n,n} \psi_n^\dagger(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; t) \hat{\mathbf{p}}_{n,a_n} \psi_n(\mathbf{x}_{n,1}, \dots, \mathbf{x}_{n,n}; t). \quad (92)$$

Finally, note that (90) can also be written in an alternative form

$$\langle p_A^\mu \rangle = \int \mathcal{D}\vec{x} \rho(\vec{x}) U_A^\mu(\vec{x}), \quad (93)$$

where

$$\rho(\vec{x}) = \Psi^\dagger(\vec{x}) \Psi(\vec{x}) \quad (94)$$

is the probability density and

$$U_A^\mu(\vec{x}) = \frac{J_A^\mu(\vec{x})}{\Psi^\dagger(\vec{x}) \Psi(\vec{x})}. \quad (95)$$

Here J_A^μ is given by an obvious generalization of (46)

$$J_A^\mu(\vec{x}) = \frac{i}{2} \Psi^\dagger(\vec{x}) \overleftrightarrow{\partial}_A^\mu \Psi(\vec{x}). \quad (96)$$

The expression (93) will play an important role in the next subsection.

4.2 Particle-trajectory interpretation

The idea of the particle-trajectory interpretation is that each particle has some trajectory $X_A^\mu(s)$, where s is an auxiliary scalar parameter that parameterizes the trajectories. Such trajectories must be consistent with the probabilistic interpretation (85). Thus, we need a velocity function $V_A^\mu(\vec{x})$, so that the trajectories satisfy

$$\frac{dX_A^\mu(s)}{ds} = V_A^\mu(\vec{X}(s)), \quad (97)$$

where the velocity function must be such that the following conservation equation is obeyed

$$\frac{\partial \rho(\vec{x})}{\partial s} + \sum_{A=1}^{\infty} \partial_{A\mu} [\rho(\vec{x}) V_A^\mu(\vec{x})] = 0. \quad (98)$$

Namely, if a statistical ensemble of particle positions in spacetime has the distribution (94) for some initial s , then (97) and (98) will provide that this statistical ensemble will also have the distribution (94) for *any* s , making the trajectories consistent with (85). The first term in (98) trivially vanishes: $\partial \rho(\vec{x})/\partial s = 0$. Thus, the condition (98) reduces to the requirement

$$\sum_{A=1}^{\infty} \partial_{A\mu} [\rho(\vec{x}) V_A^\mu(\vec{x})] = 0. \quad (99)$$

In addition, we require that the average velocity should be proportional to the average momentum (93), i.e.,

$$\int \mathcal{D}\vec{x} \rho(\vec{x}) V_A^\mu(\vec{x}) = \text{const} \times \int \mathcal{D}\vec{x} \rho(\vec{x}) U_A^\mu(\vec{x}). \quad (100)$$

In fact, the constant in (100) is physically irrelevant, because it can always be absorbed into a rescaling of the parameter s in (97). The physical 3-velocity dX_A^i/dX_A^0 , $i = 1, 2, 3$, is not affected by such a rescaling. Thus, in the rest of the analysis we fix

$$\text{const} = 1. \quad (101)$$

As a first guess, Eq. (100) with (101) suggests that one could take $V_A^\mu = U_A^\mu$. However, it does not work in general. Namely, from (94) and (95) we see that $\rho U_A^\mu = J_A^\mu$, and we have seen in (47) that J_A^μ does not need to be conserved. Instead, we have

$$\sum_{A=1}^{\infty} \partial_{A\mu} [\rho(\vec{x}) U_A^\mu(\vec{x})] = J(\vec{x}), \quad (102)$$

where $J(\vec{x})$ is some function that can be calculated explicitly whenever $\Psi(\vec{x})$ is known. So, how to find the appropriate function $V_A^\mu(\vec{x})$?

The problem of finding V_A^μ is solved in [13] for a very general case (see also [34]). Since the detailed derivation is presented in [13], here we only present the final results. Applying the general method developed in [13], one obtains

$$V_A^\mu(\vec{x}) = U_A^\mu(\vec{x}) + \rho^{-1}(\vec{x})[e_A^\mu + E_A^\mu(\vec{x})], \quad (103)$$

where

$$e_A^\mu = -\mathcal{V}^{-1} \int \mathcal{D}\vec{x}' E_A^\mu(\vec{x}'), \quad (104)$$

$$E_A^\mu(\vec{x}) = \partial_A^\mu \int \mathcal{D}\vec{x}' G(\vec{x}, \vec{x}') J(\vec{x}'), \quad (105)$$

$$G(\vec{x}, \vec{x}') = \int \frac{\mathcal{D}\vec{k}}{(2\pi)^{4N_0}} \frac{e^{i\vec{k}(\vec{x}-\vec{x}')}}{\vec{k}^2}. \quad (106)$$

Eqs. (105)-(106) provide that (103) obeys (99), while (104) provides that (103) obeys (100)-(101).

We note two important properties of (103). First, if $J = 0$ in (102), then $V_A^\mu = U_A^\mu$. In particular, since $J = 0$ for free fields in states with a definite number of particles (it can be derived for any type of particles analogously to the derivation of (48) for spinless uncharged particles), it follows that $V_A^\mu = U_A^\mu$ for such states. Second, if $\Psi(\vec{x})$ does not depend on some coordinate x_B^μ , then both $U_B^\mu = 0$ and $V_B^\mu = 0$. [To show that $V_B^\mu = 0$, note first that $J(\vec{x})$ defined by (102) does not depend on x_B^μ when $\Psi(\vec{x})$ does not depend on x_B^μ . Then the integration over dx_B^μ in (105) produces $\delta(k_B^\mu)$, which kills the dependence on x_B^μ carried by (106)]. This implies that dead particles have zero 4-velocity.

The results above show that the relativistic Bohmian trajectories are compatible with the spacetime probabilistic interpretation (85). But what about the more conventional space probabilistic interpretations (87) and (89)? *A priori*, these Bohmian trajectories are not compatible with (87) and (89). Nevertheless, as discussed in more detail in Appendix B for the 1-particle case, the compatibility between measurable predictions of the Bohmian interpretation and that of the “standard” purely probabilistic interpretation restores when the appropriate theory of quantum measurements is also taken into account.

Having established the general theory of particle trajectories by the results above, now we can discuss particular consequences.

The trajectories are determined uniquely if the initial spacetime positions $X_A^\mu(0)$ in (97), for all $\mu = 0, 1, 2, 3$, $A = 1, \dots, \infty$, are specified. In particular, since dead particles have zero 4-velocity, such particles do not really have trajectories in spacetime. Instead, they are represented by dots in spacetime, as in Fig. 2 (Sec. 2). The spacetime positions of these dots are specified by their initial spacetime positions.

Since $\rho(\vec{x})$ describes probabilities for particle creation and destruction, and since (98) provides that particle trajectories are such that spacetime positions of particles are distributed according to $\rho(\vec{x})$, it implies that particle trajectories are also consistent with particle creation and destruction. In particular, the trajectories in

spacetime may have beginning and ending points, which correspond to points at which their 4-velocities vanish (for an example, see Fig. 1). For example, the 4-velocity of the particle A vanishes if the conditional wave function $\Psi(x_A, \vec{X}')$ does not depend on x_A (where \vec{X}' denotes the actual spacetime positions of all particles except the particle A).

One very efficient mechanism of destroying particles is through the interaction with the environment, such that the total quantum state takes the form (10). The environment wave functions $(\vec{x}|E_0\rangle)$, $(\vec{x}|E_{n;i_1,\dots,i_n}\rangle)$ do not overlap, so the particles describing the environment can be in the support of only one of these environment wave functions. Consequently, the conditional wave function is described by only one of the terms in the sum (10), which effectively collapses the wave function to only one of the terms in (9). For example, if the latter wave function is $(\vec{x}|i_1, \dots, i_n\rangle)$, then it depends on only n coordinates among all x_A . All other live particles from sectors with $n' \neq n$ become dead, i.e., their 4-velocities become zero which appears as their destruction in spacetime. More generally, if the overlap between the environment wave functions is negligible but not exactly zero, then particles from sectors with $n' \neq n$ will not become dead, but their influence on the environment will still be negligible, which still provides an effective collapse to $(\vec{x}|i_1, \dots, i_n\rangle)$. Since decoherence is practically irreversible (due to many degrees of freedom involved), such an effective collapse is irreversible as well.

Another physically interesting situation is when the entanglement with the environment takes the form (12), where $|\xi\rangle$ are coherent states. In this case, the behavior of the environment can very well be described in terms of an environment that responds to a presence of classical fields. This explains how classical fields may appear at the macroscopic level, even when the microscopic ontology is described in terms of particles. Since $|\xi\rangle$ is a superposition of states with all possible numbers of particles, trajectories of particles from sectors with different numbers of particles coexist; there is an infinite number of live particle trajectories in that case.

Similarly, an entanglement of the form of (14) explains how accelerated detectors and detectors in a gravitational field may detect particles in the vacuum. For example, let us consider the case of a uniformly accelerated detector. In this case, $|0'\rangle$ corresponds to the Rindler vacuum, while $|0\rangle$ is referred to as the Minkowski vacuum [20]. The particle trajectories described by (97) are those of the Minkowski particles. The interaction between the Minkowski vacuum and the accelerated detector creates new Minkowski particles. For instance, if the detector is found in the state $|E_0\rangle$, then the Minkowski particles are in the state $|0'\rangle$, which is a squeezed state describing an infinite number of live particle trajectories. Such a view seems particularly appealing from the point of view of recently discovered renormalizable Horava-Lifshitz gravity [35] that contains an absolute time and thus a preferred definition of particles in a classical gravitational background [36].

Let us also give a few remarks on measurements of 4-momenta and 4-velocities. If $|i\rangle$ in (7) denote the 4-momentum eigenstates, then (10) describes a measurement of the particle 4-momenta. Since the 4-momentum eigenstates are also the 4-velocity eigenstates, (10) also describes a measurement of the particle 4-velocities. Thus, as

discussed also in more detail in [12], even though the Bohmian particle velocities may exceed the velocity of light, they cannot exceed the velocity of light when their velocities are measured. Instead, the effective wave function associated with such a measurement is a momentum eigenstate of the form of (49), where $k_A^2 = m^2$ for live particles and $k_A = 0$ for dead particles. This also explains why dead particles are not seen in experiments: their 3-momenta and energies are equal to zero.

Finally, we want to end this subsection with some conceptual remarks concerning the physical meaning of the parameter s . This parameter can be thought of as an evolution parameter, playing a role similar to that of the absolute Newton time t in the usual formulation of nonrelativistic Bohmian interpretation [1, 1]. To make the similarity with such a usual formulation more explicit, it may be useful to think of s as a coordinate parameterizing a “fifth dimension” that exists independently of other 4 dimensions with coordinates x^μ . However, such a 5-dimensional view should not be taken too literally. In particular, while the time t is measurable, the parameter s is not measurable.

Given the fact that s is not measurable, what is the physical meaning of the claim that particles have the distribution $\rho(\vec{x})$ at some s ? We can think of it in the following way: We always measure spacetime positions of particles at some values of s , but we do not know what these values are. Consequently, the probability density that describes our knowledge is described by $\rho(\vec{x})$ averaged over all possible values of s . However, since $\rho(\vec{x})$ does not depend on s , the result of such an averaging procedure is trivial, giving $\rho(\vec{x})$ itself.

5 Conclusion

In this paper we have extended the Bohmian interpretation of QM, such that it also incorporates a description of particle creation and destruction described by QFT. Unlike the previous attempts [10, 11] and [12, 13] to describe the creation and destruction of pointlike particles within the Bohmian interpretation, the approach of the present paper incorporates the creation and destruction of pointlike particles *automatically*, without adding any additional structure not already present in the equations that describe the continuous particle trajectories. One reason why it works is the fact that we work with a many-time wave function, so that the 4-velocity of each particle may vanish separately. Even though the many-time wave function plays a central role, we emphasize that the many-time wave function, first introduced in [33], is a natural concept when one wants to treat time on an equal footing with space, even if one does not have an ambition to describe particle creation and destruction [9]. Another, even more important reason why it works is the entanglement with the environment, which explains an effective wave function collapse into particle-number eigenstates (or some other eigenstates) even when particles are not really created or destroyed.

As a byproduct, in this paper we have also obtained many technical results that allow to represent QFT states with uncertain number of particles in terms of many-time wave functions. These results may be useful by they own, even without the Bohmian interpretation (see, e.g., [37]).

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A Spinors and coordinate transformations

At each point of spacetime, one can introduce the tetrad $e_{\bar{\alpha}}^{\mu}(x)$, which is a collection of four spacetime vectors, $\bar{\alpha} = 0, 1, 2, 3$. The bar on $\bar{\alpha}$ denotes that $\bar{\alpha}$ is not a spacetime-vector index, but only a label. By contrast, the index μ is a spacetime-vector index. The tetrad is chosen so that

$$\eta^{\bar{\alpha}\bar{\beta}} e_{\bar{\alpha}}^{\mu}(x) e_{\bar{\beta}}^{\nu}(x) = g^{\mu\nu}(x), \quad (107)$$

where $g^{\mu\nu}(x)$ is the spacetime metric and $\eta_{\bar{\alpha}\bar{\beta}}$ are components of a matrix equal to the Minkowski metric. The spacetime-vector indices are raised and lowered by $g^{\mu\nu}(x)$ and $g_{\mu\nu}(x)$, respectively, while $\bar{\alpha}$ -labels are raised and lowered by $\eta^{\bar{\alpha}\bar{\beta}}$ and $\eta_{\bar{\alpha}\bar{\beta}}$, respectively. Thus, (107) can also be inverted as

$$g^{\mu\nu}(x) e_{\mu}^{\bar{\alpha}}(x) e_{\nu}^{\bar{\beta}}(x) = \eta^{\bar{\alpha}\bar{\beta}}. \quad (108)$$

Now let $\gamma^{\bar{\alpha}}$ be the standard Dirac matrices [27]. From them we define

$$\gamma^{\mu}(x) = e_{\bar{\alpha}}^{\mu}(x) \gamma^{\bar{\alpha}}. \quad (109)$$

The spinor indices carried by matrices $\gamma^{\bar{\alpha}}$ and $\gamma^{\mu}(x)$ are interpreted as indices of the spinor representation of the *internal* group $\text{SO}(1,3)$. Thus, $\gamma^{\bar{\alpha}}$ transform as scalars under spacetime coordinate transformations. Similarly, the spinors $\psi(x)$ are also scalars with respect to spacetime coordinate transformations, while their spinor indices are indices in the internal group $\text{SO}(1,3)$. Likewise, $\psi^{\dagger}(x)$ is also a scalar with respect to spacetime coordinate transformations. It is also convenient to define the quantity

$$\bar{\psi}(x) = \psi^{\dagger}(x) \gamma^{\bar{0}}, \quad (110)$$

which is also a scalar with respect to spacetime coordinate transformations. Thus we see that the quantities

$$\bar{\psi}(x) \psi(x), \quad \psi^{\dagger}(x) \psi(x), \quad (111)$$

are both scalars with respect to spacetime coordinate transformations, and that the quantities

$$\bar{\psi}(x) \gamma^{\mu}(x) \psi(x), \quad i \psi^{\dagger}(x) \overleftrightarrow{\partial}^{\mu} \psi(x), \quad (112)$$

are both vectors with respect to spacetime coordinate transformations.

Finally, we note that the relation with the more familiar (but less general) formalism in Minkowski spacetime [27] can be established by using the fact that in flat spacetime there is a global choice of coordinates in which

$$\gamma^{\mu}(x) = \gamma^{\bar{\mu}}. \quad (113)$$

However, (113) is not a covariant expression, but is only valid in one special system of coordinates.

B Spacetime probability density and space probability density

In this Appendix we present a more careful discussion of the meaning of our probabilistic interpretation that treats time on an equal footing with space. The first subsection deals with the pure probabilistic interpretation that does not assume the existence of particle trajectories, while the second subsection deals with the Bohmian interpretation in which the existence of particle trajectories is also taken into account. For simplicity, we study the case of 1 particle only, while the generalization to many particles is straightforward.

B.1 Pure probabilistic interpretation

In a pure probabilistic interpretation of QM, one does not assume the existence of particle trajectories. Instead, the main axiom is that $\psi(x)$ determines the spacetime probability density

$$dP = |\psi(x)|^2 d^4x. \quad (114)$$

This probability describes a statistical ensemble consisting of a large number of events, where each event is an appearance of the particle at the spacetime point x . Now, if we pick up a subensemble consisting of all events x that have the same time coordinate $x^0 = t$, then the distribution in this subensemble is given by the conditional probability

$$dP_{(3)} = \frac{|\psi(\mathbf{x}, t)|^2 d^3x}{N_t}, \quad (115)$$

where the normalization factor $N_t = \int d^3x |\psi(\mathbf{x}, t)|^2$ is equal to the marginal probability that the particle from the initial ensemble will have $x^0 = t$. (In fact, when $\psi(x)$ is a superposition of plane waves with positive frequencies only, then N_t does not depend on t [38].)

A more formal way to understand (114) is as follows. We start from the identity

$$\psi(x) = \int d^4x' \psi(x') \delta^4(x' - x). \quad (116)$$

Formally, it is convenient to use a discrete notation

$$\int d^4x' \rightarrow \sum_{x'}, \quad \psi(x') \rightarrow c_{x'}, \quad \delta^4(x' - x) \rightarrow \psi_{x'}(x), \quad (117)$$

where the function $\psi_{x'}(x)$ can be thought of as an eigenstate of the operator x with the eigenvalue x' . Thus, (116) can be written as

$$\psi(x) = \sum_{x'} c_{x'} \psi_{x'}(x). \quad (118)$$

The probability that the particle will be found in the eigenstate $\psi_{x'}(x)$ is equal to $|c_{x'}|^2$. Recalling (117), this again leads to (114).

Yet another way to understand (114) and (115) is through the theory of *ideal* quantum measurements. The measured particle with the position x entangles with the measuring apparatus described by a pointer position y . Thus, instead of (118) we have the total wave function

$$\Psi(x, y) = \sum_{x'} c_{x'} \psi_{x'}(x) E_{x'}(y), \quad (119)$$

where $E_{x'}(y)$ are normalized apparatus states that do not overlap in the y -space. The main axiom (114) now includes y as well, so that (114) generalizes to $dP = |\Psi(x, y)|^2 d^4x dy$. Therefore, the probability that y will have a value from the support of $E_{x'}(y)$ is equal to $|c_{x'}|^2$, which again leads to (114). Eq. (115) emerges from (114) as a conditional probability within the statistical ensemble of measurement outcomes.

Of course, a real experiment is not ideal. If the departure from ideality is small, then the distributions of measurement outcomes (114) and (115) are good approximations. However, a real experiment may in fact be far from being ideal. In particular, in a real experiment the functions $\psi_{x'}(x)$ (which are localized in both space and time) in (119) may be replaced by functions $\psi_{\mathbf{x}'}(\mathbf{x}, t)$ which are well localized in space, but very extended in time. In such an experiment, (119) is replaced by an entanglement of the form

$$\Psi(\mathbf{x}, t, y) = \sum_{\mathbf{x}'} c_{\mathbf{x}'}(t) \psi_{\mathbf{x}'}(\mathbf{x}, t) E_{\mathbf{x}'}(y). \quad (120)$$

For definiteness, $\psi_{\mathbf{x}'}(\mathbf{x}, t)$ may be taken to be time independent and proportional to $\delta^3(\mathbf{x}' - \mathbf{x})$ for $t \in [t_0 - \Delta t/2, t_0 + \Delta t/2]$, but vanishing for other values of t . Now the measuring apparatus measures the space position \mathbf{x} of the particle very well, but time t at which the particle attains this position remains uncertain with uncertainty equal to Δt . Instead of (115), the measured distribution of particle positions is now

$$dP_{(3)} = \rho(\mathbf{x}) d^3x, \quad (121)$$

where

$$\rho(\mathbf{x}) = \frac{\int_{t_0 - \Delta t/2}^{t_0 + \Delta t/2} dt |\psi(\mathbf{x}, t)|^2}{N}, \quad (122)$$

and N is the normalization factor chosen such that $\int d^3x \rho(\mathbf{x}) = 1$. Namely, the probability that y will have a value from the support of $E_{\mathbf{x}}(y)$ is given by (121).

We also stress that the difference between (115) and (121) is irrelevant in most practical situations. Namely, in actual experiments one usually deals with wave functions that are well approximated by energy eigenstates with a trivial time dependence proportional to e^{-iEt} . Consequently, $|\psi|^2$ is almost independent on t , which makes (115) and (121) practically indistinguishable.

B.2 Bohmian interpretation

Now let us assume that particles have trajectories

$$\frac{dX^\mu(s)}{ds} = v^\mu(X(s)), \quad (123)$$

where $v^\mu(x)$ is such that

$$\partial_\mu(|\psi|^2 v^\mu) = 0. \quad (124)$$

Are such trajectories compatible with the probabilistic predictions studied in Sec. B.1? Clearly, they are compatible with (114) because

$$\frac{\partial|\psi|^2}{\partial s} + \partial_\mu(|\psi|^2 v^\mu) = 0. \quad (125)$$

But are they compatible with (115)? To answer this question, it is useful to eliminate s from (123) by introducing velocities

$$\frac{dX^i}{dx^0} = \frac{v^i}{v^0} \equiv u^i, \quad \text{for } i = 1, 2, 3. \quad (126)$$

In general, (124) implies that

$$\frac{\partial|\psi|^2}{\partial t} + \partial_i(|\psi|^2 u^i) \neq 0. \quad (127)$$

Inequality (127) shows that, *a priori*, (123) is not compatible with (115) (see also [5]).

Nevertheless, the compatibility restores when the theory of quantum measurements is also taken into account. Namely, we extend (123) and (124) such that $Y(s)$ also satisfies a Bohmian equation of motion compatible with $dP = |\Psi(x, y)|^2 d^4x dy$. For ideal quantum measurements, (119) implies that the probability that Y will take a value from the support of $E_{x'}(y)$ is equal to $|c_{x'}|^2$. This means that the statistics of measurement outcomes is given by (114). Thus, (115) emerges from (114) as a conditional probability within the statistical ensemble of measurement outcomes. Similarly, in a more realistic measurement based on (120), the probability that Y will take a value from the support of $E_{\mathbf{x}}(y)$ is given by (121).

Thus we see that the space probability density (115) does not necessarily need to be correct. Instead, the space probability density depends on how exactly it is measured. However, the important points are (i) that the space probability density can in principle be predicted from the fundamental axiom (114) when the measuring procedure is well defined, and (ii) that the probabilistic predictions of the Bohmian interpretation agree with those of the “standard” (pure probabilistic) interpretation for ideal measurements based on (119), as well as for more realistic measurements based on (120).

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