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Quantum states for primitive ontologists

A case study

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Abstract Under so-called primitive ontology approaches, in fully describing the history of a quantum system, one thereby attributes interesting properties to regions of spacetime. Primitive ontology approaches, which include some varieties of Bohmian mechanics and spontaneous collapse theories, are interesting in part because they hold out the hope that it should not be too difficult to make a connection between models of quantum mechanics and descriptions of histories of ordinary macroscopic bodies. But such approaches are dualistic, positing a quantum state as well as ordinary material degrees of freedom. This paper lays out and compares some options that primitive ontologists have for making sense of the quantum state.

Keywords Wave-function · Ontology · Bohm

1 Introduction

It is a truth universally acknowledged that an approach to understanding quantum mechanics is acceptable only if it allows us to construct models of the sort of experiments that provide evidence in favour of the theory.

One thing that this maxim requires of any acceptable account of quantum mechanics is that it predict that experiments have determinate outcomes. Suppose that our laboratory apparatus is set up so that: (i) when it is fed a spin-x up electron, Chester (a walrus) ends up spending the afternoon basking

Dedicated to the memory of Ayveq, a great pinniped.

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on a rock at the Coney Island Aquarium; (ii) when it is fed a spin-x down electron, Chester ends up spending the afternoon ransacking the fish stalls of Brighton Beach. Experience with such experimental setups leads us to expect that if our apparatus were fed an electron in a non-trivial superposition of spin-x states, then it would be one of these two results that obtained—rather than some strange alternative in which Chester had no determinate location or employment for the afternoon. The problem of explicating, supplementing, or altering the bare formalism of quantum mechanics so as to assure that this expectation is fulfilled is the *measurement problem.*[1]

Another requirement that follows from our maxim is that any acceptable account of quantum mechanics must support models of situations in which ordinary objects like walruses, stars, and instrument dials behave in familiar ways. Let us just focus on one aspect of this challenge: providing truth conditions for assignments of ordinary properties to macro-objects. Let us call this the *macro-object problem.* Of course, the actual goal here is not to cook up truth conditions for specific claims about Chester and the like. Rather we aim to feel in the quantum case, as we do in the classical case, that we understand in broad outline what a model of a walrus basking on a rock would look like.

Approaches to understanding quantum mechanics divide on the question whether in fully describing the history of a quantum system one typically thereby attributes an interesting family of properties to regions of spacetime. Approaches that give an affirmative answer to this question are sometimes called *primitive ontology* approaches.[2] These include versions of Bohmian mechanics in which specifying the history of a quantum system involves specifying the continuous spacetime trajectories of Bohmian particles and versions of spontaneous collapse theories in which by specifying the history of the quantum state one determines the (physically real) mass density distribution on spacetime or picks out the (ontologically distinguished) set of spacetime points corresponding to the collapse events associated with each particle.[3] These stand in contrast to many-worlds approaches, under which in specifying the history of a quantum system one specifies a solution of the Schrödinger equation (which does not in general suffice to underwrite any interesting facts about the location of a quantum system or of its parts) and those variants of the Bohmian and of spontaneous collapse approaches under which, ontologically speaking, all of the action is in the system's configuration space rather than in spacetime.[4]

The macro-object problem is far more tractable for primitive ontology approaches than for others: in taking the specification of a history of a quantum

[1]Rather than attempting to solve the measurement problem, one can attempt to evade or deflate it by denying that quantum states are in the business of representing how things are in the world. For a sophisticated version of this neo-instrumentalist strategy, see Bub and Pitowsky (2010).

[2]The source of this terminology is Goldstein (1998).

[3]For a survey of these options considered as primitive ontology approaches, see Allori et al. (2008).

[4]For discussion of aspects of the macro-object problem for these approaches, see Wallace (2003) and Albert (1996).

system to involve the attribution of properties to regions of spacetime primitive ontologists provide themselves with resources of the same sort employed by classical theories in providing truth conditions for macro-claims.

This is a real advantage of primitive ontology approaches. But it is important not to lose sight of the fact that extant primitive ontology approaches are dualistic—the primitive ontology is not the whole story.[5] Under a primitive ontology approach, in describing the state or history of a quantum system, one attributes properties to regions of spacetime. We can think of this as the specification of the behaviour of the material degrees of freedom of the system. These material degrees of freedom belong to some more or less familiar category, and so count (by the standards of philosophy of quantum mechanics) as ontologically unproblematic. But the material degrees of freedom posited by (extant) primitive ontology approaches are not dynamically autonomous: in order to write down the equations governing their evolution, one has to assign each system a wave-function evolving according to the Schrödinger equation or one of its relatives.[6] So in addition to the material degrees of freedom that allow primitive ontology approaches to handle the macro-object problem, one must posit further degrees of freedom. Collectively these are known as the *quantum state* and are represented by the wave-function.

Essentially every approach to understanding quantum mechanics must face sooner or later the questions concerning the physical content and ontological status of the quantum state. Primitive ontology approaches provide a controlled and structured context in which these questions can be faced squarely—so they seem to me to provide a promising starting point for thinking about the quantum state. My project here is an exploratory one. I will examine: (i) some of the ontological options for making sense of the wave-function within primitive ontology approaches; and (ii) some of the connections between these options and various technical and interpretative questions. In order to keep the discussion manageable, I will focus on a single primitive ontology approach, Bohmian mechanics.[7]

2 Bohmian mechanics

2.1 Formalism

Consider the Bohmian theory of n spinless particles living in Euclidean three-space. The dynamical variables of the theory include both configuration

[5] On this point, see, e.g., Allori et al. (2008, p. 363) and Tumulka (2007, p. 3260). For an attempt to eliminate the wave-function from spontaneous collapse theories, see Dowker and Herbauts (2005).

[6] On this point, see, e.g., Maudlin (2008, pp. 173 f.) and Tumulka (2006, Section 1).

[7] Each of the options canvassed below can be found in the literature, except the dispositionalist account.

variables for Bohmian particles and a wave-function encoding the quantum state of the system.

The classical configuration space for this system is $Q := \mathbb{R}^{3n}$—a point $q = (q_1, \ldots, q_n)$ of Q specifies the position q_i in Euclidean space of each of the n particles. A *wave-function* is a complex-valued function ψ on Q such that

$$|| \psi ||^2 := \int_Q \psi(x)\psi^*(x)\, dx$$

is finite and positive. We introduce an equivalence relation on wave-functions by stipulating that two wave-functions are equivalent if they disagree only on a subset of Q of measure zero or if one is a complex multiple of the other.[8] Equivalent wave-functions correspond to the same quantum state—and under standard approaches quantum states are identified with equivalence classes of wave-functions under this equivalence relation.[9] The evolution in time of the wave-function (and hence of the quantum state) is determined by the Schrödinger equation.

An instantaneous state of the Bohmian particles is given by specifying specifying a point $q := (q_1, \ldots, q_n) \in Q$. So one specifies a history of the Bohmian particles by specifying a curve $q(t)$ in Q. The dynamics for the Bohmian particles is given by the *Bohmian law of motion* that singles out the dynamically allowed $q(t)$:

$$\frac{d}{dt} q_k(t) = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_k \psi}{\psi}(q(t)), \qquad (1)$$

where m_k is the mass of the kth particle, ψ is the wave-function of the system, and ∇_k is the gradient corresponding to the coordinates of q_k.[10]

In addition to driving the dynamics of the Bohmian particles, the wave-function of a system plays another role: the real-valued function

$$p(q) := \frac{| \psi(q) |^2}{|| \psi ||^2}$$

on Q is a probability distribution central to the empirical content of the theory.[11]

2.2 Interpretation

This formalism raises a number of interpretative puzzles. I will mention three of the most obvious. (1) How should we think of the quantum state

[8] I.e., we look for the weakest equivalence relation that satisfies these stipulations.

[9] We will see a non-standard approach below, under which inequivalent wave-functions are sometimes taken to correspond to the same quantum state.

[10] At this point, various subtleties arise concerning smoothness and domains of definition; see Berndl et al. (1995), Hall (2004), and Grübl and Penz (2011).

[11] Equivalent wave functions will not in general agree about $p(q)$, but they will agree about the integral of $p(q)$ over any region of Q.

represented by the wave-function? (2) How should we think of the particle degrees of freedom of the theory? (3) How should we understand the role of the probability distribution p associated with the wave-function?

Our interest here is in the first of these three questions. But it will be helpful to pause just for a moment over the other two. There are two ways of thinking of the particle degrees of freedom of Bohmian mechanics. Under what is probably the standard view, one thinks of the instantaneous particle state $q = (q_1, \ldots, q_n)$ as assigning each of n Bohmian particles a position in Euclidean three-space, so that a history $q(t)$ of the particles assigns a worldline in spacetime to each particle. Under an alternative view, one thinks of the particle state q as describing the location of a single "world-particle" in configuration space (understood as the basic arena of physics), and thinks of a history of the particle state as singling out a sort of worldline for the world-particle in the system's configuration space.[12] Only the first of these two options amounts to a primitive ontology approach to understanding quantum mechanics, so it will be our primary focus below.

As for the probability density, one popular approach is to view it as grounding a notion of *typicality* for particle states—telling us which regions of the system's configuration space correspond to particle states that are in some sense to be expected, and which correspond to particle states whose instantiation would be unexpected.[13]

And what of the quantum state? A natural idea is to assimilate it to a classical ontological category. Three possibilities that come to mind are: take the wave-function as corresponding to an object; take the wave-function as corresponding to a law; take the wave-function as corresponding to a property.

ψ as object When one rummages through the familiar categories of objects, one comes across one outstanding candidate to provide a suitable ontological home for the wave-function: the category of fields.

In classical physics, a configuration of a field is thought of as an assignment of a property to each point of space or spacetime. In familiar cases, the property in question determines the force that a particle of a given kind would feel if located at that point—e.g., the Lorentz force law specifies the force that would be felt by a particle with charge e and velocity v located at point x in terms of the electromagnetic field at x. A field is naturally represented by a map from points of space or spacetime to some target space that parameterizes the family of properties up for grabs.

The wave-function of a single-particle quantum system is a map that associates complex numbers with points of space. So in this case, it is tempting to

[12]For this approach, see Albert (1996, Section 1) and Loewer (1996, Section I). The approach is often said to have its roots in remarks of Bell—see the remark quoted in fn. 17 below.
[13]For discussion and references, see Callender (2007).

think of the wave-function as corresponding to a field on the physical space in which the system is located, analogous to the gravitational and electric fields except that the family of properties up for grabs in the quantum case is parameterized by the complex numbers.[14]

Unfortunately, this picture cannot be simply carried over to the $n > 1$ case: a wave-function is a map that assigns a complex number to an n-tuple of points of three-space—and so can be thought of as representing a field on three-space only in the $n = 1$ case.

Still, it is natural to take our inspiration from the single-particle case, and to look for an ontology that reduces to a field ontology in that special case. One option is to think of a wave-function as a field on configuration space:

> The sorts of physical objects that wave functions *are* ... are (plainly) *fields*—which is to say that they are the sorts of objects whose states one specifies by specifying the values of some set of numbers at each point in the space where they live, the sorts of objects whose state one specifies (in *this* case) by specifying the values of *two* [real] numbers ... at every point in the universe's so-called *configuration space*.[15]

But there *are* (plainly) other *options*. One is to introduce the notion of a *multi-field*, a configuration of which assigns properties to sets or fusions of n points, and to view n-particle wave-functions as corresponding to multi-fields on ordinary three-space rather than to fields on the much larger configuration space of the system.[16]

Neither of these approaches is ideal. Part of their motivation lies in the thought that the wave-function plays a role in the Bohmian law of motion analogous to the role that the electromagnetic field plays in the Lorentz force law.[17] But, even in the one-particle case, the wave-function is not really *that* similar to a field. (i) Part of the role of fields like the electromagnetic field (and one important reason for taking them to be physical in nature) is that they ensure conservation of energy and momentum when bodies act on one another at a distance. The wave-function plays no such role. (ii) In paradigmatic field theories, the field both acts on and is acted upon by matter (charged bodies not only obey the Lorentz force law, but also provide the source terms in Maxwell's

[14]Of course, our real interest is in the quantum state, not the wave-function. In order to see quantum states as analogous to fields, we should first generalize the notion of a field so that it tells us not about the assignment of properties to points of space, but about something like the ratio of integrals of such assignments over regions of space.

[15]Albert (1996, p. 278); see also Loewer (1996, Section I).

[16]For this approach, see Forrest (1988, Chapter 5).

[17] Something along these lines appears to stand behind Bell's dictum that "*No one can understand* [Bohmian mechanics] *until he is willing to think of* ψ *as a real objective field rather than just as a 'probability amplitude'. Even though it propagates not in* 3*-space but in* 3*N-space*" (Bell 1987, p. 128).

equations).[18] But the wave-function is entirely indifferent to the doings of the Bohmian particles (e.g., the Schrödinger dynamics takes no account of which of the dynamically possible histories of the Bohmian particles is actual). (iii) Further, the wave-function transforms under boosts in a manner quite different from that of classical fields. Let ϕ be a (real or complex) scalar field on spacetime, and ψ be a wave-function. And let x and t be the coordinates of a given point with respect to one set of inertial coordinates on spacetime, and x' and t' be coordinates of the same point with respect to a second set of inertial coordinates, moving with non-zero velocity v relative to the first. Then $\phi(x, t) = \phi(x', t')$ but in general $\psi(x, t) \neq \psi(x', t')$.[19]

There are some interesting differences between our two field-based approaches.

If we think of the wave-function as a field on configuration space, then it enjoys a sort of locality (it is determined by specifying its value at each point of the appropriate space) that is alien to the multi-field approach.[20] But it is not obvious that this is a significant advantage in this context. The kind of locality that is bought by taking configuration space rather than ordinary physical space as the arena of physics is not of course the sort of locality that was hankered after by those critical of quantum mechanics. And while it is sometimes argued that in order to avoid flirtation with incomprehensibility a physical theory must involve "local beables" (i.e., localizable physical degrees of freedom) in the case at hand the Bohmian particles already satisfy this demand.[21]

We can think of each of the approaches under consideration as arising via generalization from the field interpretation of single-particle Bohmian mechanics. Under the multi-field approach, we generalize an essentially technical notion, thinking of a field as a special case of the multi-field. In the case of the wave-function as field-on-configuration-space approach, on the other hand, we are required to think that in addition to the three-dimensional space in which the Bohmian particles live and move and have their being there is also a physically real zillion-dimensional space for the wave-function.[22] This rather more startling direction of generalization brings with it a number of interesting questions—about why the world should appear to be (only) three-dimensional to us, and about the range of circumstances under which this appearance would

[18]Well-known mathematical difficulties stand in the way of implementing this sort of reciprocity—but in the case of Maxwell's theory, rigourous models have been constructed for the case of finite charged bodies. See Spohn (2004).

[19]The two wave-functions will differ by a phase factor that depends on x, t, v, and the mass of the particle under consideration. See, e.g., Ballentine (1998, Section 4.3).

[20]Loewer (1996, Section I).

[21]For discussion of this point, see Maudlin (2007, Section 3).

[22]A view of this kind is espoused by Dorr (2009, *Finding ordinary objects in some quantum worlds*, unpublished manuscript)—whose reasons for rejecting the multi-field approach turn on the subtle question of whether or not there are fundamental non-symmetric relations.

persist.[23] But it is perhaps natural to be wary of taking these questions on board, given the relative dearth of motivation offered for taking configuration space to be an ontologically primary arena of physics—since the corresponding question faced by alternative approaches (Why does reality appear to be three-dimensional when it is three-dimensional?) are *prima facie* less daunting.[24]

ψ as law Consider once again Eq. 1, the Bohmian law of motion. The interpretative avenue just considered took this equation to be analogous to the Lorentz force law—with the wave-function playing a role in the Bohmian law of motion analogous to that played by the electromagnetic field in the Lorentz force law. As was noted above, this analogy is in some ways forced: e.g., because while in classical electrodynamics one aims to formulate a theory in which matter acts on the field as well as being acted upon by it, within Bohmian mechanics the wave-function determines the motion of the Bohmian particles without being in any way affected by their motion.

According to one school of thought, the mistake here is basing the guiding analogy on the wrong equation of classical mechanics.[25] Consider the Hamiltonian treatment of a system of n Newtonian particles subject to Newton's law of universal gravitation. The *phase space* of such a system consists of ordered pairs (q, p), with q encoding the instantaneous position of the particles and p encoding the instantaneous momentum of the particles. A history of the system is represented by a trajectory $(q(t), p(t))$ in the system's phase space. The *Hamiltonian* for this system is the real-valued function $H(q, p)$ on the system's phase space that assigns to each state (q, p) the sum of the system's kinetic and potential energy when in that state. The dynamically allowed $(q(t), p(t))$ are determined by *Hamilton's equations*:

$$\frac{d}{dt}q(t) = \frac{\partial H}{\partial p}(q(t), p(t)) \qquad \frac{d}{dt}p(t) = -\frac{\partial H}{\partial q}(q(t), p(t)).$$

[23]The problem is even more striking on the picture on which the particle configuration variable of the Bohmian theory describes the location of a world-particle in configuration space rather than the locations of many particles in three-space:

> the space [wave functions] *live* in, and (therefore) the space *we* live in, the space in which any realistic understanding of quantum mechanics is necessarily going to depict the history of the world as *playing itself out* (if space is the right name for it ...) is *configuration*-space. And whatever impression we have to the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional space-time) is somehow flatly illusory. (Albert 1996, p. 277)

[24]Some readers may well feel that, if true, the claim made by Bell in the passage quoted in fn. 17 above provides ample motivation to prefer the field-in-configuration-space picture to the multi-field-in-three-space picture. Others will be dubious that facts concerning human mathematical processing can carry much weight in the present context.

[25]For the ideas sketched in this paragraph, see Dürr et al. (1997, Section 12) and Goldstein and Teufel (2001, Section 12.6).

No one thinks it strange that H determines the motions the classical system without depending in any way on these motions—since H is not intended to represent a substantial entity (like a field) but rather is a mathematical means of encoding *nomological* structure. It is possible to think of the wave-function in similar terms, as representing an aspect of the laws of the theory rather than as representing *stuff* additional to the Bohmian particles. On this reading the Schrödinger equation becomes a sort of meta-law governing the evolution in time of that aspect of the nomological structure encoded in the wave-function.

The suggestion that the wave-function is in some sense nomological is fascinating. But it appears to have two consequences that will give pause to many.[26] (1) The wave-function of a system is in general time-dependent. This seems to imply that the nomological structure of the theory is itself changing in time or, perhaps more palatably, that it involves some sort of temporal indexing (i.e., we can think of a solution to the Schrödinger equation as determining a tenseless law-proposition that is temporally indexed in the sense that the sort of motion it decrees for a given configuration of Bohmian particles depends on the time at which that configuration obtains). (2) Some laws are more necessary than others—intuitively at least, the nomological structure encoded in the Schrödinger equation would still have held, even if the part of the nomological structure encoded in the wave-function had been different, but not vice versa—so the wave-function is in a sense more contingent, more like an initial condition, than is the Schrödinger equation.

Advocates of the wave-function-as-law approach have proposed a bold research program that, if carried to fruition, would show that this approach is not in fact committed to grades of modality or time-indexed laws.[27] The guiding idea is that one expects that any fundamental theory of the entire universe would have to be a quantum theory of gravity. And one has independent reason to believe that the quantum state should be time-independent in such a theory.[28] One assumes that in a theory of quantum gravity the appearance of time-dependence would result from restriction of attention to subsystems. And many hope that such a theory would feature a unique nomologically permitted wave-function (picked out by the Hartle-Hawking no-boundary proposal or some similar condition).

In the absence of such a theory, however, what should we think about the wave-function-as-law approach to applied to theories like Bohmian mechanics? Here one is stuck with time-indexed laws and with grades of physical necessity.

The first of these consequences seems to be to provide the more daunting obstacle. Most philosophical accounts of laws of nature allow that temporally-indexed laws are possible. But one is used to regarding temporal-indexing as

[26]These are briefly noted in Dürr et al. (1997, Section 12).

[27]See Dürr et al. (1997, Section 13 f.) and Goldstein and Teufel (2001, Section 12.6).

[28]For discussion and references, see, e.g., Belot and Earman (2001) or Butterfield and Isham (1999).

a remote possibility—certainly not one forced upon us by any of our central physical theories. And this does much, I think, to undermine the salience of the analogy between the role of the wave-function ψ in the Bohmian law of motion and the role of the classical Hamiltonian H in Hamilton's equations (note H is time-independent in paradigmatic decent physical theories).

The need to introduce grades of modality should not, I think, constitute the same sort of sticking point. It is true that on some accounts of laws of nature, it makes no sense of speak of some laws as being more necessary than others.[29] But then there are other accounts of laws of nature on which such gradations arise quite naturally.[30] And something very like a recognition of grades of necessity seems to be implicit in much contemporary philosophical discussion of statistical physics. For example: classical statistical mechanics can be thought of as adding to the basic Newtonian dynamical framework postulates that (i) stipulate that the initial state of the universe was a low-entropy state, and (ii) give a probability density over eligible initial micro-states. In some ways these postulates function like laws.[31] But they are of course quite different in character from the laws of the underlying dynamical framework. In particular, it seems very strange to say that the initial state of the world is constrained by law: intuitively, imagining a possible world that is always in a high-entropy state is not the same sort of exercise as imagining a possible world in which the basic dynamical laws are violated.[32] One natural response to these observations would be to think that while the characteristic postulates of statistical mechanics are nomological and do enjoy a degree of physical necessity, they are *less* necessary than the fundamental dynamical laws.[33]

[29] Consider, e.g., Lewis's version of the best-system account (see, e.g., Lewis 1999, pp. 39–43 and 231–236). On this account the laws are the consequences of those generalizations concerning the pattern of instantiation of perfectly natural properties that provide the best (=strongest-simplest) description of the actual world. Because lawhood is, roughly speaking, a global property of systems of axioms, it makes little sense to single out any member of such a system as being more or less necessary than any other.

[30] Consider here the account of Lange (2009), on which the laws are distinguished from accidents via their (collective) exhibition of a certain sort of stability under a certain wide range of counterfactual suppositions. As Lange emphasizes, the set of laws may include a subset distinguished by its (collective) stability under a yet wider range of counterfactual suppositions—such a subset enjoys a form of necessity weaker than metaphysical necessity but somewhat stronger than that it exhibited by the full set of laws. For example: on this account, the symmetry principles of classical mechanics are somewhat more necessary than the other laws of classical mechanics (see Lange 2009, Sections 1.4, 1.9, and 3.5).

[31] This is a prominent theme in Sklar (1993) and in Albert (2000).

[32] It is sometimes suggested that the best-system account of laws be generalized so that the characteristic postulates of statistical mechanics (which are not generalizations, and so do not count as laws under Lewis's version of the account) will come out as laws at our world (for this suggestion, see Loewer 2001 or Callender 2004). This leads others to object that it is a weakness of best-system approach that it lends itself so readily to a blurring of the distinction between between laws and initial conditions (see Roberts 2008, Section 1.6).

[33] Other considerations arising out of quantum statistical mechanics suggest a similar sort of picture (see Ruetsche 2011).

So, for many philosophers at least, the fact the the wave-function-as-law approach involves grades of physical necessity need not be much of an impediment—although one might still of course worry that the fact that there is so little imaginative resistance when one is asked to imagine the wave-function being different from what it actually is suggests that the present approach accords very ill with how we are used to thinking about quantum mechanics.

ψ as property Consider again the Hamiltonian treatment of a system of classical particles. The instantaneous dynamical state of such a system is specified by assigning each of its particles a position and a momentum. The system's phase space is an abstract space the points of which correspond to the possible dynamical states of the system. A possible history of the system is represented by a trajectory through this phase space—with the dynamically possible trajectories being determined via Hamilton's equations. Since each point of the system's phase space corresponds to a position-and-momentum property the system might have, we can think of the phase space as a whole as representing a determinable family of properties, with each point of the space corresponding to an individual determinate property.

There is some temptation to think of quantum mechanics in the same terms.[34] We can think of the space of quantum states (here, the space of equivalence classes of wave-functions) as parameterizing a determinable family of properties, so that in specifying the wave-function of a quantum system we specify the determinate member of this family that the system instantiates at a given time. A possible history of the system corresponds to a trajectory through the space of quantum states—with the physically possible histories being determined via the system's Schrödinger equation.

This strategy seems promising—so far as it goes. But it is unlikely to satisfy unless we can be told more about the bearer of the property in question and about what distinguishes the various properties up for grabs.[35] However, Bohmian mechanics provides a context in which this interpretative strategy can be filled out in a clear, contentful way.

Consider, by way of motivation, my cowardice. It is among my dispositional properties, determining, how fast (and in what direction) I would move in various possible situations in which I might find myself.

The wave-function can be thought of in similar terms. The Bohmian law of motion (Eq. 1 above) is in effect a recipe that takes as input a system's wave-function ψ and gives as output a function that assigns to each possible particle configuration $q = (q_1, \ldots, q_n) \in Q$ the velocities that the particles would have were the system in that configuration and were the quantum state of the system given by ψ. So we can think of the Bohmian law of motion as a rule via

[34] For a suggestion along these lines, see Monton (2006).

[35] Contrast with the classical case, where we can say that the bearer of the property is a system of particles and that each property under consideration is an assignment to each particle of a position and a momentum.

which the wave-function ψ of a system determines a complicated dispositional property Φ of the system—the dispositional property that determines how fast (and in what direction) each of the particles would move for each possible configuration of the system of particles. Correspondingly we can think of the Bohmian law of motion as giving us a rule via which a solution $\psi(t)$ of the system's Schrödinger equation determines a one-parameter family Φ_t of such properties, one for each instant of time.

Let's see what happens if we take Φ_t rather than $\psi(t)$ as encoding full information concerning the quantum state. That is, let us explore an interpretation of Bohmian mechanics under which the complete history of a quantum system is specified by specifying for each time t a configuration $q(t)$ of the particles together with the dispositional property Φ_t that tells us, for each possible configuration of the system of particles at t, what the velocity of each particle would be were that configuration actual. On this interpretation, all we have are particles and properties of (systems of) particles—at each time, each particle has a mass and a position and the system of particles as a whole has a further dispositional property. Let us call this approach the *dispositionalist interpretation* of Bohmian mechanics.

A few worries about this interpretation come readily to mind. Many will feel uncomfortable with the dispositions invoked, either because they are bare dispositions (not somehow grounded in categorical properties) or because they are holistic properties of the system of particles, not analysable into properties of the individual particles. But neither of these worries need dissuade an advocate of the dispostitionalist approach. If one likes, one can always posit a new sort of categorical property suitable to ground an otherwise bare disposition together with a physical or metaphysical necessity that accomplishes the grounding. And Bohmians, at any rate, are liable to accept that quantum mechanics ineluctably involves some form of holism.[36]

To my mind the real worry about this approach turns on questions of detail concerning the content of the quantum state, rather than on generic features of its metaphysics.

The natural way to mathematically represent an instantaneous dispostional property Φ of the sort under consideration is via a vector field X on Q (such a vector field assigns a tangent vector to each point in Q—and can be understood as determining, for every possible configuration of the system of particles, what the velocity of each particle would be in that configuration). Correspondingly, the natural way to encode a history Φ_t of such dispositional properties is via a time-dependent vector field X_t on Q (i.e., a function that assigns a vector field on Q to each time t). We can think of Eq. 1 above as defining a map $\mathscr{B} : \psi(t) \mapsto X_t$ that associates with each (suitably smooth) solution of the system's Schrödinger equation a time-dependent vector field X_t

[36]Holistic physical properties have other uses as well—e.g., in relationalist theories of motion historical (see Lodge 2003, pp. 284 f.) and contemporary (see Belot 2001, Section VI).

that encodes the history Φ_t.[37] On the interpretation currently under discussion, it is X_t rather than $\psi(t)$ that perspicously represents the history of the system: two solutions of the Schrödinger equation correspond to the same history of the quantum state if and only if they are mapped to the same X_t by $\mathscr{B}$.

Clearly, if solutions $\psi(t)$ and $\phi(t)$ of a system's Schrödinger equation are equivalent (i.e., for each t_1, $\psi(t_1)$ and $\phi(t_1)$ are equivalent), then they are mapped to the same X_t by $\mathscr{B}$ and hence correspond to the same history of the quantum state under our dispositonalist interpretation. But can inequivalent $\psi(t)$ and $\phi(t)$ be mapped to the same X_t by $\mathscr{B}$? That is, does the dispositionalist interpretation individuate (histories of) quantum states in a non-standard fashion?

Yes. Consider that perennial nuisance for Bohmians, the particle in a box.[38] Let us focus on the simplest case: a single Bohmian particle restricted to the unit interval $[0, 1] \subset \mathbb{R}$. Consider the wave-functions:

$$\phi_n(x) = \sin(\pi n x), \quad n = 1, 2, \ldots .$$

The Bohmian law of motion tells us that if the particle is in a state corresponding to one of these wave-functions at time t, then the particle is at rest at t, no matter where it is in the box.[39] If the particle we are considering is free, then each ϕ_n is in fact an energy eigenstate. This means that if the system's wave-function is given by ϕ_n at one time, then it is given by (a wave-function equivalent to) ϕ_n at all times—and so induces a history of dispositions according to which the particle would be at rest at each moment, no matter where it were located in its box. In this way we find an infinite family of inequivalent solutions to our system's Schrödinger equation, each corresponding to the same history Φ_t of dispostional properties.[40]

This is dismaying. Under the standard approach, different ϕ_n correspond to different energetic states of the system—the larger n is, the more dangerous it is to stick your hand in a box containing a particle in state ϕ_n. Likewise, under

[37] Again, on the issue of smoothness, see Berndl et al. (1995, esp. Section 4.4).

[38] For the role that this system played in early criticisms of Bohmian mechanics, see Myrvold (2003). The harmonic oscillator is another simple, charismatic quantum system that could be used to illustrate the following points (it is easier to write down the relevant family of functions for the particle in a box).

[39] Note that this shows that the system is not deterministic if we take Φ and the particle positions to exhaust the instantaneous state and attempt to mimic the dynamics of ordinary Bohmian mechanics. We have seen that ϕ_1 and ϕ_2 determine the same instantaneous dispositional property Φ_0. But specifying that at time t_0 the particle is in configuration q_0 and that the dispositional property of the system is Φ_0 will not in general suffice to determine the particle's position at future times: we could choose a Hamiltonian for our system that has ϕ_1 but not ϕ_2 as an eigenstate; then in ordinary Bohmian mechanics, we would find that our initial data (q_0, Φ_0) is consistent with the particle remaining eternally in q_0 or with it moving around (depending whether the initial quantum state is given by ϕ_1 or by ϕ_2). Of course, one would hope that determinism would be secured if one were to include one or more 'time-derivatives of Φ' among the dynamical variables that describe tinstantaneous states.

[40] Note, though, that a nontrivial superposition of two such solutions would correspond to a quite different dispositional history.

the standard approach, distinct ϕ_n in general disagree as to the probability of finding the particle in any given region of space—e.g., according to ϕ_1, it is quite typical for the particle to be located near the midpoint of the box, while according to ϕ_2 this is rather atypical. These distinctions are effaced under the dispositionalist interpretation: all solutions corresponding to energy eigenstates of the free particle in a box correspond to the same history of dispositions—and so cannot differ in regard to danger or typicality.

Observations of this sort may seem to all but scupper the dispositionalist interpretation: for it may well appear that in taking dispositional histories rather than histories of the quantum state to be fundamental, the dispositionalist interpretation discards a great deal of the essential physical content of quantum mechanics.

But there is hope for the dispositionalist interpretation. We have seen that the following claim is not in general true:

> Solutions $\psi(t)$ and $\phi(t)$ of a system's Schrödinger equation induce the same time-dependent vector field X_t on the system's configuration space only if they are equivalent.

But it is natural to conjecture that this claim is true for generic solutions (i.e., that if fails at most for a tiny subset of a system's solutions).[41] If this conjecture is correct, then it is only in very special cases that one throws away any physical information in passing from a solution $\psi(t)$ to the corresponding family Φ_t of velocity dispositions that it induces via the Bohmian law of motion—since one can in fact (ordinarily) reconstruct knowledge of the history of the ordinary quantum state from knowledge of the history of dispositional properties.

Only in those special cases in which such reconstruction is impossible will one have thrown away information about probabilities or values of measurable physical quantities in taking Φ_t rather than $\psi(t)$ as basic. And it would appear to be possible to bite the bullet concerning such special cases by saying that if it is impossible to reconstruct from knowledge of the dispositional history a family of solutions of the Schrödinger equation, all members of which agree about typicality or energy (or some other quantity), then in such cases there are no determinate facts regarding typicality, or energy, or whatever. And from the point of view of a primitive ontologist who views Bohmian mechanics as being fundamentally about the particles and their motions this may in fact appear quite natural.[42]

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[41] Sheldon Goldstein, private communication (November 2008).

[42] It is important to keep in mind here that it is only when we are treat the free particle in a box or the harmonic oscillator as closed systems that problems arise: if such systems interact with others, then there is no reason to expect the composite system to exhibit difficulties of the kind under discussion. (This point is the analog of the standard Bohmian reply to Einstein's worry about Bohmian mechanics. *Worry*—If the quantum state of a particle in a box is an eigenstate of energy, then the particle is stationary—this is unphysical. *Reply*—Any decent model of the measurement of the momentum of such a system will give predictions agreeing with those of standard quantum mechanics.)

3 Inconclusive conclusion

We have in effect considered four suggestions for explicating the nature of the wave-function in (primitive ontology) Bohmian mechanics.

Multi-field: Think of the wave-function as a sort of generalized field (assigning properties to *n*-tuples of points of space) living on ordinary three-dimensional physical space.
Field: Think of the wave-function as a physical field (i.e., as an assignment of properties to points of space) on a physically real space of a very high dimension.
Law: Think of the wave-function as representing nomological rather than material structure.
Property: Think of the wave-function as a device for encoding the velocity dispositions of the Bohmian particles.

Each of these involves reliance on some metaphysical apparatus that will not universally be held to be above reproach. The viability of the second, third, and fourth options is entangled with difficult-looking questions. How can we explain the fact that the world appears to be merely three-dimensional, if in fact there is a also physically real space of enormous dimensionality? Can one show that in the case of a fundamental quantum theory, taking the wave function as a law doesn't require any significant revision of our notion of law? Can one show that in typical cases nothing is lost in speaking of velocity dispositions rather than the wave-function—and satisfy oneself with living without what is lost in the other cases? So there is a sense in which the first option is the safest—it is ready to go off the shelf, as it were. Correlatively, there is a sense in which it is the least interesting.

What should one make of all this? That depends on one's views about the aims and methods of interpreting physical theories. On the orthodox quasi-Quinean view, one should posit the simplest ontology consistent with a conservative preference for minimal revisions in our overall theory—and should expect this procedure to lead to the truth (so the apposite terms of praise are 'plausible,' 'well-supported' and the like). According to the heresy promulgated by van Fraassen, there is no reason to believe simplicity and familiarity to be marks of truth in matters of ontology—so we should see the aim of interpretation as the delineation and investigation of alternatives in the service of increasing understanding, rather than in the search for truth (so the apposite terms of praise are 'interesting,' 'well-motivated,' and the like).[43] In the case at hand, it looks to me like the orthodox should tentatively judge the multi-field approach to be the most plausible—it is straightforward and requires relatively little revision. But my own views tend towards heresy.[44]

[43] See, e.g., van Fraassen (2007, p. 358).
[44] See Belot (2011, Appendix A).

So I think it it is worthwhile to think about other interesting, well-motivated approaches.

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